Problem A.1
All parts of the problem are solved using the relation

$$V_{\rm rms} = \sqrt{4kTRB}$$

where

$$\begin{aligned} k &= 1.38 \times 10^{-23} \text{ J/K} \\ B &= 30 \text{ MHz} = 3 \times 10^{7} \text{ Hz} \end{aligned}$$

a. For $R = 10,000$ ohms and $T = T_0 = 290$ K

$$\begin{aligned} V_{\rm rms} &= \sqrt{4\,(1.38 \times 10^{-23})\,(290)\,(10^4)\,(3 \times 10^7)} \\ &= 6.93 \times 10^{-5} \text{ V rms} \\ &= 69.3\ \mu\text{V rms} \end{aligned}$$

b. $V_{\rm rms}$ is smaller than the result in part (a) by a factor of $\sqrt{10} = 3.16$. Thus

$$V_{\rm rms} = 21.9\ \mu\text{V rms}$$

c. $V_{\rm rms}$ is smaller than the result in part (a) by a factor of $\sqrt{100} = 10$. Thus

$$V_{\rm rms} = 6.93\ \mu\text{V rms}$$

d. Each answer becomes smaller by factors of 2, $\sqrt{10} = 3.16$, and 10, respectively.

Problem A.2
Use

$$I = I_s \left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

We want $I > 20I_s$ or $\exp\left(\frac{eV}{kT}\right) - 1 > 20$.

a. At $T = 290$ K, $\frac{e}{kT} = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{23} \times 290} \cong 40$, so we have $\exp(40V) > 21$ giving

$$\begin{aligned} V &> \frac{\ln(21)}{40} = 0.0761 \text{ volts} \\ i_{\rm rms}^2 &\cong 2eIB \simeq 2eBI_s \exp\left(\frac{eV}{kT}\right) \\ \text{or} \quad \frac{i_{\rm rms}^2}{B} &= 2eI_s \exp\left(\frac{eV}{kT}\right) \\ &= 2\left(1.6 \times 10^{-19}\right)\left(1.5 \times 10^{-5}\right)\exp(40 \times 0.0761) \\ &= 1.0075 \times 10^{-22} \text{ A}^2/\text{Hz} \end{aligned}$$

b. If $T = 90$ K, then $\frac{e}{kT} \cong 129$, and for $I > 20I_s$, we need $\exp(129V) > 21$ or

$$V > \frac{\ln(21)}{129} = 2.36 \times 10^{-2} \text{ volts}$$

Thus

$$\begin{aligned}\frac{i_{\text{rms}}^2}{B} &= 2\left(1.6\times10^{-19}\right)\left(1.5\times10^{-5}\right)\exp\left(129\times0.0236\right)\\ &= 1.0079\times10^{-22}\ \text{A}^2/\text{Hz}\end{aligned}$$

approximately as before.

Problem A.3

a. Use Nyquist's formula to get the equivalent circuit of R_{eq} in parallel with R_L, where R_{eq} is given by

$$R_{\text{eq}} = \frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3}$$

The noise equivalent circuit consists of a rms noise voltage, V_{eq}, in series with R_{eq} and a rms noise voltage, V_L, in series with with R_L with these series combinations being in parallel. The equivalent noise voltages are

$$\begin{aligned}V_{\text{eq}} &= \sqrt{4kTR_{\text{eq}}B}\\ V_L &= \sqrt{4kTR_LB}\end{aligned}$$

The rms noise voltages across the parallel combination of R_{eq} and R_L, by using superposition and voltage division, are

$$V_{01} = \frac{V_{\text{eq}}R_L}{R_{\text{eq}}+R_L} \text{ and } V_{02} = \frac{V_LR_{\text{eq}}}{R_{\text{eq}}+R_L}$$

Adding noise powers to get V_0^2 we obtain

$$\begin{aligned}V_0^2 &= \frac{V_{\text{eq}}^2R_L^2}{\left(R_{\text{eq}}+R_L\right)^2}+\frac{V_L^2R_{\text{eq}}^2}{\left(R_{\text{eq}}+R_L\right)^2}\\ &= \frac{\left(4kTB\right)R_LR_{\text{eq}}}{R_{\text{eq}}+R_L}\\ &= 4kTB\frac{R_LR_3\left(R_1+R_2\right)}{R_1R_3+R_2R_3+R_1R_L+R_2R_L+R_3R_L}\end{aligned}$$

Note that we could have considered the parallel combination of R_3 and R_L as an equivalent load resistor and found the Thevenin equivalent. Let

$$R_{||} = \frac{R_3R_L}{R_3+R_L}$$

The Thevenin equivalent resistance of the whole circuit is then

$$\begin{aligned}R_{\text{eq}2} &= \frac{R_{||}\left(R_1+R_2\right)}{R_{||}+R_1+R_2} = \frac{\frac{R_3R_L}{R_3+R_L}\left(R_1+R_2\right)}{\frac{R_3R_L}{R_3+R_L}+R_1+R_2}\\ &= \frac{R_LR_3\left(R_1+R_2\right)}{R_1R_3+R_2R_3+R_1R_L+R_2R_L+R_3R_L}\end{aligned}$$

and the mean-square output noise voltage is now

$$V_0^2 = 4kTBR_{\text{eq2}}$$

which is the same result as obtained before.

b. With $R_1 = 2000\ \Omega,\ R_2 = R_L = 300\ \Omega$, and $R_3 = 500\ \Omega$, we have

$$\begin{aligned}
\frac{V_0^2}{B} &= 4kTB\frac{R_L R_3 (R_1 + R_2)}{R_1R_3 + R_2R_3 + R_1R_L + R_2R_L + R_3R_L} \\
&= \frac{4\left(1.38 \times 10^{-23}\right)(290)(300)(500)(2000 + 300)}{2000(500) + 300(500) + 2000(300) + 300(300) + 500(300)} \\
&= 2.775 \times 10^{-18}\ \text{V}^2/\text{Hz}
\end{aligned}$$

Problem A.4

Find the equivalent resistance for the $R_1,\ R_2,\ R_3$ combination and set R_L equal to this to get

$$R_L = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3}$$

Problem A.5

Using Nyquist's formula, we find the equivalent resistance looking back into the terminals with V_{rms} across them. It is

$$\begin{aligned}
R_{\text{eq}} &= 50\ \text{k} \parallel 20\ \text{k} \parallel (5\ \text{k} + 10\ \text{k} + 5\ \text{k}) \\
&= 50\ \text{k} \parallel 20\ \text{k} \parallel 20\ \text{k} \\
&= 50\ \text{k} \parallel 10\ \text{k} \\
&= \frac{(50\ \text{k})(10\ \text{k})}{50\ \text{k} + 10\ \text{k}} \\
&= 8,333\ \Omega
\end{aligned}$$

Thus

$$\begin{aligned}
V_{\text{rms}}^2 &= 4kTR_{\text{eq}}B \\
&= 4\left(1.38 \times 10^{-23}\right)(400)(8333)\left(2 \times 10^6\right) \\
&= 3.68 \times 10^{-10}\ \text{V}^2
\end{aligned}$$

which gives

$$V_{\text{rms}} = 19.18\ \mu\text{V rms}$$

Problem A.6

To find the noise figure, we first determine the noise power due to a source at the output, then due to the source and the network, and take the ratio of the latter to the former. Initally assume unmatched conditions. The results are

$$V_0^2\Big|_{\text{due to } R_S\text{, only}} = \left(\frac{R_2 \parallel R_L}{R_S + R_1 + R_2 \parallel R_L}\right)^2 (4kTR_SB)$$

$$\begin{aligned} V_0^2\Big|_{\text{due to } R_1 \text{ and } R_2} &= \left(\frac{R_2 \parallel R_L}{R_S + R_1 + R_2 \parallel R_L}\right)^2 (4kTR_1B) \\ &+ \left(\frac{R_L \parallel (R_1 + R_S)}{R_2 + (R_1 + R_S) \parallel R_L}\right)^2 (4kTR_2B) \end{aligned}$$

$$\begin{aligned} V_0^2\Big|_{\text{due to } R_S\text{, } R_1 \text{ and } R_2} &= \left(\frac{R_2 \parallel R_L}{R_S + R_1 + R_2 \parallel R_L}\right)^2 [4kT(R_S + R_1)B] \\ &+ \left(\frac{R_L \parallel (R_1 + R_S)}{R_2 + (R_1 + R_S) \parallel R_L}\right)^2 (4kTR_2B) \end{aligned}$$

The noise figure is (after some simplification)

$$F = 1 + \frac{R_1}{R_S} + \left(\frac{R_L \parallel (R_1 + R_S)}{R_2 + (R_1 + R_S) \parallel R_L}\right)^2 \left(\frac{R_S + R_1 + R_2 \parallel R_L}{R_2 \parallel R_L}\right)^2 \frac{R_2}{R_S}$$

In the above,

$$R_a \parallel R_b = \frac{R_aR_b}{R_a + R_b}$$

Note that the noise due to R_L has been excluded because it belongs to the next stage. Since this is a matching circuit, we want the input matched to the source and the output matched to the load. Matching at the input requires that

$$R_S = R_{\text{in}} = R_1 + R_2 \parallel R_L = R_1 + \frac{R_2R_L}{R_2 + R_L}$$

and matching at the output requires that

$$R_L = R_{\text{out}} = R_2 \parallel (R_1 + R_S) = \frac{R_2(R_1 + R_S)}{R_1 + R_2 + R_S}$$

Next, these expressions are substituted back into the expression for F. After some simplification, this gives

$$F = 1 + \frac{R_1}{R_S} + \left(\frac{2R_L^2R_S(R_1 + R_2 + R_S)/(R_S - R_1)}{R_2^2(R_1 + R_S + R_L) + R_L^2(R_1 + R_2 + R_S)}\right)^2 \frac{R_2}{R_S}$$

Note that if $R_1 >> R_2$ we then have matched conditions of $R_L \cong R_2$ and $R_S \cong R_1$. Then, the noise figure simplifies to

$$F = 2 + 16\frac{R_1}{R_2}$$

Note that the simple resistive pad matching circuit is very poor from the standpoint of noise. The equivalent noise temperature is found by using

$$\begin{aligned} T_e &= T_0 (F - 1) \\ &= T_0 \left[1 + 16 \frac{R_1}{R_2} \right] \end{aligned}$$

Problem A.7

a. The important relationships are

$$F_l = 1 + \frac{T_{e_l}}{T_0}$$

$$T_{e_l} = T_0 (F_l - 1)$$

$$T_{e_0} = T_{e_1} + \frac{T_{e_2}}{G_{a_1}} + \frac{T_{e_3}}{G_{a_1} G_{a_2}}$$

Completion of the table gives

Ampl. No.	F	T_{e_i}	G_{a_i}
1	not needed	300 K	10 dB = 10
2	6 dB	864.5 K	30 dB = 1000
3	11 dB	3360.9 K	30 dB = 1000

Therefore,

$$\begin{aligned} T_{e_0} &= 300 + \frac{864.5}{10} + \frac{3360.9}{(10)(1000)} \\ &= 386.8 \text{ K} \end{aligned}$$

Hence,

$$\begin{aligned} F_0 &= 1 + \frac{T_{e_0}}{T_0} \\ &= 2.33 = 3.68 \text{ dB} \end{aligned}$$

b. With amplifiers 1 and 2 interchanged

$$\begin{aligned} T_{e_0} &= 864.5 + \frac{300}{10} + \frac{3360.9}{(10)(1000)} \\ &= 865.14 \text{ K} \end{aligned}$$

This gives a noise figure of

$$\begin{aligned} F_0 &= 1 + \frac{865.14}{290} \\ &= 3.98 = 6 \text{ dB} \end{aligned}$$

c. See part (a) for the noise temperatures.

d. For $B = 50$ kHz, $T_S = 1000$ K, and an overall gain of $G_a = 10^7$, we have, for the configuration of part (a)

$$\begin{aligned} P_{na,\text{ out}} &= G_a k \left(T_0 + T_{e_0}\right) B \\ &= 10^7 \left(1.38 \times 10^{-23}\right)(1000 + 386.8)\left(5 \times 10^4\right) \\ &= 9.57 \times 10^{-9} \text{ watts} \end{aligned}$$

We desire

$$\frac{P_{sa,\text{ out}}}{P_{na,\text{ out}}} = 10^4 = \frac{P_{sa,\text{ out}}}{9.57 \times 10^{-9}}$$

which gives

$$P_{sa,\text{ out}} = 9.57 \times 10^{-5} \text{ watts}$$

For part (b), we have

$$\begin{aligned} P_{na,\text{ out}} &= 10^7 \left(1.38 \times 10^{-23}\right)(1000 + 865.14)\left(5 \times 10^4\right) \\ &= 1.29 \times 10^{-8} \text{ watts} \end{aligned}$$

Once again, we desire

$$\frac{P_{sa,\text{ out}}}{P_{na,\text{ out}}} = 10^4 = \frac{P_{sa,\text{ out}}}{1.29 \times 10^{-8}}$$

which gives

$$P_{sa,\text{ out}} = 1.29 \times 10^{-4} \text{ watts}$$

and

$$P_{sa,\text{ in}} = \frac{P_{sa,\text{ out}}}{G_a} = 1.29 \times 10^{-11} \text{ watts}$$

Problem A.8

a. The noise figure of the cascade is

$$F_{\text{overall}} = F_1 + \frac{F_2 - 1}{G_{a_1}} = L + \frac{F-1}{(1/L)} = LF$$

b. For two identical attenuator-amplifier stages

$$F_{\text{overall}} = L + \frac{F-1}{(1/L)} + \frac{L-1}{(1/L)\,L} + \frac{F-1}{(1/L)\,L\,(1/L)} = 2LF - 1 \approx 2LF, \quad L >> 1$$

c. Generalizing, for N stages we have

$$F_{\text{overall}} \approx NFL$$

Problem A.9

a. The data for this problem is

Stage	F_i	G_i
1 (preamp)	2 dB = 1.58	G_1
2 (mixer)	8 dB = 6.31	1.5 dB = 1.41
3 (amplifier)	5 dB = 3.16	30 dB = 1000

The overall noise figure is

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

which gives

$$5 \text{ dB} = 3.16 = 1.58 + \frac{6.31 - 1}{G_1} + \frac{3.16 - 1}{1.41 G_1}$$

or

$$\begin{aligned} 3.16 - 1.58 &= \frac{6.31 - 1}{G_1} + \frac{3.16 - 1}{1.41 G_1} \\ \text{or } G_1 &= \frac{5.31}{1.58} + \frac{2.16}{(1.41)\,(1.58)} = 4.33 = 6.37 \text{ dB} \end{aligned}$$

b. First, assume that $G_1 = 15$ dB $= 31.62$. Then

$$\begin{aligned} F &= 1.58 + \frac{6.31 - 1}{31.62} + \frac{3.16 - 1}{(1.41)\,(31.62)} \\ &= 1.8 = 2.54 \text{ dB} \end{aligned}$$

Then

$$\begin{aligned} T_{es} &= T_0(F-1) \\ &= 290(1.8-1) \\ &= 230.95 \text{ K} \end{aligned}$$

and

$$\begin{aligned} T_{\text{overall}} &= T_{es} + T_a \\ &= 230.95 + 300 \\ &= 530.95 \text{ K} \end{aligned}$$

Now use G_1 as found in part (a):

$$\begin{aligned} F &= 3.16 \\ T_{es} &= 290(3.16-1) = 626.4 \text{ K} \\ T_{\text{overall}} &= 300 + 626.4 = 926.4 \text{ K} \end{aligned}$$

c. For $G_1 = 15 \text{ dB} = 31.62,\ G_a = (31.62)(1.41)(1000) = 4.46 \times 10^4$. Thus

$$\begin{aligned} P_{na,\text{ out}} &= G_a k T_{\text{overall}} B \\ &= \left(4.46 \times 10^4\right)\left(1.38 \times 10^{-23}\right)(530.95)\left(10^7\right) \\ &= 3.27 \times 10^{-6} \text{ watts} \end{aligned}$$

For $G_1 = 6.37 \text{ dB} = 4.33,\ G_a = (4.33)(1.41)(1000) = 6.11 \times 10^3$. Thus

$$\begin{aligned} P_{na,\text{ out}} &= \left(6.11 \times 10^3\right)\left(1.38 \times 10^{-23}\right)(926.4)\left(10^7\right) \\ &= 7.81 \times 10^{-7} \text{ watts} \end{aligned}$$

Note that for the second case, we get less noise power out even wth a larger T_{overall}. This is due to the lower gain of stage 1, which more than compensates for the larger input noise power.

d. A transmission line with loss $L = 2$ dB connects the antenna to the preamp. We first find T_S for the transmission line/preamp/mixer/amp chain:

$$F_S = F_{\text{TL}} + \frac{F_1 - 1}{G_{\text{TL}}} + \frac{F_2 - 1}{G_{\text{TL}} G_1} + \frac{F_3 - 1}{G_{\text{TL}} G_1 G_2},$$

where

$$G_{\text{TL}} = 1/L = 10^{-2/10} = 0.631 \text{ and } F_{\text{TL}} = L = 10^{2/10} = 1.58$$

Assume two cases for G_1: 15 dB and 6.37 dB. First, for $G_1 = 15 \text{ dB} = 31.6$, we have

$$\begin{aligned} F_S &= 1.58 + \frac{1.58 - 1}{0.631} + \frac{6.31 - 1}{(0.631)(31.6)} + \frac{3.16 - 1}{(0.631)(31.6)(1.41)} \\ &= 2.84 \end{aligned}$$

This gives

$$T_S = 290\,(2.84 - 1) = 534 \text{ K}$$

and

$$T_{\text{overall}} = 534 + 300 = 834 \text{ K}$$

Now, for $G_1 = 6.37$ dB $= 4.33$, we have

$$\begin{aligned} F_S &= 1.58 + \frac{1.58 - 1}{0.631} + \frac{6.31 - 1}{(0.631)\,(4.33)} + \frac{3.16 - 1}{(0.631)\,(4.33)\,(1.41)} \\ &= 5.00 \end{aligned}$$

This gives

$$T_S = 290\,(5.00 - 1) = 1160 \text{ K}$$

and

$$T_{\text{overall}} = 1160 + 300 = 1460 \text{ K}$$

Problem A.10

a. (a) Using

$$P_{na,\text{ out}} = G_a k T_S B = \left(10^8\right)\left(1.38 \times 10^{-23}\right)(1800)\left(3 \times 10^6\right)$$

with the given values yields

$$P_{na,\text{ out}} = 7.45 \times 10^{-5} \text{ watts}$$

b. We want

$$\frac{P_{sa,\text{ out}}}{P_{na,\text{ out}}} = 10^5$$

or

$$P_{sa,\text{ out}} = \left(10^5\right)\left(7.45 \times 10^{-5}\right) = 7.45 \text{ watts}$$

This gives

$$\begin{aligned} P_{sa,\text{ in}} &= \frac{P_{sa,\text{ out}}}{G_a} = \frac{7.45}{10^8} = 7.45 \times 10^{-8} \text{ watts} \\ &= -71.28 \text{ dBW} = -41.28 \text{ dBm} \end{aligned}$$

Problem A.11

a. For $\Delta A = 1$ dB, $Y = 1.259$ and the effective noise temperature is

$$T_e = \frac{600 - (1.259)(300)}{1.259 - 1} = 858.3 \text{ K}$$

For $\Delta A = 1.5$ dB, $Y = 1.413$ and the effective noise temperature is

$$T_e = \frac{600 - (1.413)(300)}{1.413 - 1} = 426.4 \text{ K}$$

For $\Delta A = 2$ dB, $Y = 1.585$ and the effective noise temperature is

$$T_e = \frac{600 - (1.585)(300)}{1.585 - 1} = 212.8 \text{ K}$$

b. These values can be converted to noise figure using

$$F = 1 + \frac{T_e}{T_0}$$

With $T_0 = 290$ K, we get the following values: (1) For $\Delta A = 1$ dB, $F = 5.98$ dB; (2) For $\Delta A = 1.5$ dB, $F = 3.938$ dB; (3) For $\Delta A = 2$ dB, $F = 2.39$ dB.

Problem A.12

a. Using the data given, we can determine the following:

$$\begin{aligned} \lambda &= 0.039 \text{ m} \\ \left(\frac{\lambda}{4\pi d}\right)^2 &= -202.4 \text{ dB} \\ G_T &= 39.2 \text{ dB} \\ P_T G_T &= 74.2 \text{ dBW} \end{aligned}$$

This gives

$$P_S = -202.4 + 74.2 + 6 - 5 = -127.2 \text{ dBW}$$

b. Using $P_n = kT_eB$ for the noise power, we get

$$\begin{aligned} P_n &= 10\log_{10}\left[kT_0\left(\frac{T_e}{T_0}\right)B\right] \\ &= 10\log_{10}[kT_0] + 10\log_{10}\left(\frac{T_e}{T_0}\right) + 10\log_{10}(B) \\ &= -174 + 10\log_{10}\left(\frac{1000}{290}\right) + 10\log_{10}\left(10^6\right) \\ &= -108.6 \text{ dBm} \\ &= -138.6 \text{ dBW} \end{aligned}$$

c.

$$\begin{aligned}\left(\frac{P_S}{P_n}\right)_{\text{dB}} &= -127.2 - (-138.6) \\ &= 11.4 \text{ dB} \\ &= 10^{1.14} = 13.8 \text{ ratio}\end{aligned}$$

d. Assuming the SNR $= z = E_b/N_0 = 13.8$, we get the results for various digital signaling techniques given in the table below:

Modulation type	Error probability
BPSK	$Q\left(\sqrt{2z}\right) = 7.4 \times 10^{-8}$
DPSK	$\frac{1}{2}e^{-z} = 5.06 \times 10^{-7}$
Noncoh. FSK	$\frac{1}{2}e^{-z/2} = 5.03 \times 10^{-4}$
QPSK	Same as BPSK