1.1

1.1 The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

$$F = C_D \rho V^2 A/2$$

or

$$C_D = 2F/\rho V^2 A, \quad \text{where} \quad F \doteq MLT^{-2}$$
$$\rho \doteq ML^{-3}$$
$$V \doteq LT^{-1}$$
$$A \doteq L^2$$

Thus,

$$C_D \doteq (MLT^{-2})/[(ML^{-3})(LT^{-1})^2(L^2)] = M^0 L^0 T^0$$

Hence, C_D is dimensionless.

4

1.2 The Mach number is a dimensionless ratio of the velocity of an object in a fluid to the speed of sound in the fluid. For an airplane flying at velocity V in air at absolute temperature T, the Mach number Ma is,

$$\mathrm{Ma} = \frac{V}{\sqrt{kRT}},$$

where k is a dimensionless constant and R is the specific gas constant for air. Show that Ma is dimensionless.

SOLUTION:

We denote the dimension of temperature by Θ and use Newton's second law to get $F = ML/T^2$. Then

$$[M] = \frac{\left(\frac{L}{T}\right)}{\sqrt{(1)\left(\frac{FL}{M\Theta}\right)\Theta\left(\frac{ML}{T^2 F}\right)}} = \frac{\left(\frac{L}{T}\right)}{\sqrt{\frac{L^2}{T^2}}}$$

or

$$[M] = [1].$$

1.3

1.3 Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table 1.1: **(a)** volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

(a) volume $\doteq \underline{\underline{L^3}}$

(b) acceleration = time rate of change of velocity

$$\doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$$

(c) mass $\doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

mass $\doteq \underline{\underline{FL^{-1}T^{2}}}$

(d) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(e) work = force $\times$ distance

$$\doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$

work $\doteq \underline{\underline{ML^2T^{-2}}}$

1.4

1.4 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: **(a)** angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

(a) angular velocity $= \dfrac{\text{angular displacement}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

(b) energy ~ capacity of body to do work

Since work = force × distance,

$$\text{energy} \doteq \underline{\underline{FL}}$$

or with $F \doteq MLT^{-2}$

$$\text{energy} \doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^2T^{-2}}}$$

(c) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(d) power = rate of doing work $\doteq \dfrac{FL}{T} \doteq \underline{\underline{FLT^{-1}}}$

$$\doteq (MLT^{-2})(L)(T^{-1}) \doteq \underline{\underline{ML^2T^{-3}}}$$

(e) pressure $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

$$\doteq (MLT^{-2})(L^{-2}) \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

1.5

1.5 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: **(a)** frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

(a) $\text{frequency} = \dfrac{\text{cycles}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

(b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$$\text{stress} \doteq \frac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

(c) $\text{strain} = \dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \underline{\underline{L^0}}$ (dimensionless)

(d) $\text{torque} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

$$\doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^2T^{-2}}}$$

(e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

$$\doteq (MLT^{-2})(L) \doteq \underline{\underline{ML^2T^{-2}}}$$