Exam

Name_____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the system of equations.

1) $x_1 - x_2 + 3x_3 = -8$
    $2x_1 \qquad + x_3 = 0$
    $x_1 + 5x_2 + x_3 = 40$

    A) $(0, -8, -8)$ B) $(-8, 0, 0)$ C) $(0, 8, 0)$ D) $(8, 8, 0)$

1) _____

2) $x_1 + 3x_2 + 2x_3 = 11$
    $4x_2 + 9x_3 = -12$
    $x_3 = -4$

    A) $(1, -4, 6)$ B) $(1, 6, -4)$ C) $(-4, 1, 6)$ D) $(-4, 6, 1)$

2) _____

3) $x_1 - x_2 + 8x_3 = -107$
    $6x_1 \qquad + x_3 = 17$
    $3x_2 - 5x_3 = 89$

    A) $(-5, 8, 13)$ B) $(5, -8, -13)$ C) $(-5, -8, 13)$ D) $(5, 8, -13)$

3) _____

4) $4x_1 - x_2 + 3x_3 = 12$
    $2x_1 \qquad + 9x_3 = -5$
    $x_1 + 4x_2 + 6x_3 = -32$

    A) $(2, -7, -1)$ B) $(2, -7, 1)$ C) $(2, 7, -1)$ D) $(2, 7, 1)$

4) _____

5) $x_1 + x_2 + x_3 = 6$
    $x_1 \qquad - x_3 = -2$
    $x_2 + 3x_3 = 11$

    A) $(0, 1, 2)$ B) $(-1, 2, -3)$ C) $(1, 2, 3)$ D) No solution

5) _____

6) $x_1 + x_2 + x_3 = 7$
    $x_1 - x_2 + 2x_3 = 7$
    $5x_1 + x_2 + x_3 = 11$

    A) $(4, 2, 1)$ B) $(4, 1, 2)$ C) $(1, 4, 2)$ D) $(1, 2, 4)$

6) _____

7) $x_1 - x_2 + x_3 = 8$
    $x_1 + x_2 + x_3 = 6$
    $x_1 + x_2 - x_3 = -12$

    A) $(2, -1, 9)$ B) $(-2, -1, -9)$ C) $(-2, -1, 9)$ D) $(2, -1, -9)$

7) _____

8) $5x_1 + 2x_2 + x_3 = -11$
    $2x_1 - 3x_2 - x_3 = 17$
    $7x_1 + x_2 + 2x_3 = -4$

    A) $(-3, 0, 4)$ B) $(3, 0, -4)$ C) $(0, -6, 1)$ D) $(0, 6, -1)$

8) _____

9) $7x_1 + 7x_2 + x_3 = 1$
   $x_1 + 8x_2 + 8x_3 = 8$
   $9x_1 + x_2 + 9x_3 = 9$

   A) $(1, -1, 1)$ B) $(-1, 1, 1)$ C) $(0, 1, 0)$ D) $(0, 0, 1)$

9) _____

10) $2x_1 + x_2 \quad = 0$
    $x_1 - 3x_2 + x_3 = 0$
    $3x_1 + x_2 - x_3 = 0$

    A) No solution B) $(0, 0, 0)$ C) $(1, 0, 0)$ D) $(0, 1, 0)$

10) _____

Determine whether the system is consistent.

11) $x_1 + x_2 + x_3 = 7$
    $x_1 - x_2 + 2x_3 = 7$
    $5x_1 + x_2 + x_3 = 11$

    A) No B) Yes

11) _____

12) $5x_1 + 2x_2 + x_3 = -11$
    $2x_1 - 3x_2 - x_3 = 17$
    $7x_1 + x_2 + 2x_3 = -4$

    A) Yes B) No

12) _____

13) $4x_1 - x_2 + 3x_3 = 12$
    $2x_1 \quad + 9x_3 = -5$
    $x_1 + 4x_2 + 6x_3 = -32$

    A) Yes B) No

13) _____

14) $2x_1 + x_2 \quad = 0$
    $x_1 - 3x_2 + x_3 = 0$
    $3x_1 + x_2 - x_3 = 0$

    A) Yes B) No

14) _____

15) $x_1 + x_2 + x_3 = 6$
    $x_1 \quad - x_3 = -2$
    $x_2 + 3x_3 = 11$

    A) Yes B) No

15) _____

16) $x_1 - x_2 + 4x_3 = 15$
    $-4x_1 + 4x_2 - 16x_3 = 4$
    $x_1 + 4x_2 + x_3 = 0$

    A) Yes B) No

16) _____

17) $x_1 + x_2 + x_3 = 7$
    $x_1 - x_2 + 2x_3 = 7$
    $2x_1 \quad + 3x_3 = 15$
      A) Yes   B) No

17) _____

18) $x_1 + 3x_2 + 2x_3 = 11$
        $4x_2 + 9x_3 = -12$
    $x_1 + 7x_2 + 11x_3 = -11$
      A) Yes   B) No

18) _____

19) $5x_1 + 2x_2 + x_3 = -11$
    $2x_1 - 3x_2 - x_3 = 17$
    $7x_1 - x_2 \quad = 12$
      A) Yes   B) No

19) _____

20) $5x_2 \quad + x_4 = -11$
  $x_1 + x_2 + 6x_3 - x_4 = 15$
  $5x_1 \quad + x_3 + 6x_4 = 16$
  $x_1 + x_2 + 3x_3 \quad = 8$
      A) Yes   B) No

20) _____

Determine whether the matrix is in echelon form, reduced echelon form, or neither.

21) $\begin{bmatrix} 1 & 2 & 5 & -7 \\ 0 & 1 & -4 & 9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

21) _____

     A) Neither   B) Reduced echelon form   C) Echelon form

22) $\begin{bmatrix} 1 & 4 & 5 & -7 \\ 0 & 1 & -4 & -6 \\ 0 & 2 & 1 & 6 \end{bmatrix}$

22) _____

     A) Reduced echelon form   B) Neither   C) Echelon form

23) $\begin{bmatrix} 1 & 4 & 5 & -7 \\ 6 & 1 & -4 & 8 \\ 0 & 5 & 1 & 6 \end{bmatrix}$

23) _____

     A) Echelon form   B) Reduced echelon form   C) Neither

24) $\begin{bmatrix} 1 & 0 & 0 & -7 \\ 2 & 1 & 0 & -2 \\ 0 & 5 & 1 & 2 \end{bmatrix}$

24) _____

     A) Reduced echelon form   B) Echelon form   C) Neither

25) $\begin{bmatrix} 1 & 6 & 2 & -7 \\ 0 & 1 & -4 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 25) _____

    A) Neither   B) Reduced echelon form   C) Echelon form

26) $\begin{bmatrix} 1 & 0 & -3 & -5 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 26) _____

    A) Neither   B) Reduced echelon form   C) Echelon form

27) $\begin{bmatrix} 1 & -5 & 3 & 4 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 27) _____

    A) Reduced echelon form   B) Echelon form   C) Neither

Use the row reduction algorithm to transform the matrix into echelon form or reduced echelon form as indicated.

28) Find the echelon form of the given matrix. 28) _____

$\begin{bmatrix} 1 & 4 & -2 & 3 \\ -3 & -11 & 9 & -5 \\ 2 & 2 & 5 & -1 \end{bmatrix}$

A) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 27 & 0 \end{bmatrix}$
   B) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & -6 & 9 & -7 \end{bmatrix}$
   C) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 27 & 17 \end{bmatrix}$
   D) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 15 & -1 \end{bmatrix}$

29) Find the reduced echelon form of the given matrix. 29) _____

$\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 2 & 5 & -4 & -1 & 4 \\ -3 & -9 & 9 & 2 & 10 \end{bmatrix}$

A) $\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$
         B) $\begin{bmatrix} 1 & 4 & -5 & 0 & -6 \\ 0 & 1 & -2 & 0 & -8 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 3 & 0 & 26 \\ 0 & 1 & -2 & 0 & -8 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$
         D) $\begin{bmatrix} 1 & 0 & 0 & 0 & 26 \\ 0 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$

The augmented matrix is given for a system of equations. If the system is consistent, find the general solution. Otherwise state that there is no solution.

30) $\begin{bmatrix} 1 & -5 & -1 \\ 0 & 0 & 7 \end{bmatrix}$

30) _____

    A) $(-1, 7)$ B) No solution   C) $x_1 = -1 + 5x_2$ D) $x_1 = -1 + 5x_2$

                                                                      $x_2 = 7$ x_2 is free

                                                                      x_3 is free

31) $\begin{bmatrix} 1 & 2 & -3 & -6 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

31) _____

    A) $x_1 = -20 + 11x_3$ B) $x_1 = -6 - 2x_2 + 3x_3$

           $x_2 = 7 - 4x_3$ x_2 is free

           $x_3 = 2$ x_3 is free

    C) $x_1 = -20 + 11x_3$ D) No solution

           $x_2 = 7 - 4x_3$

           x_3 is free

32) $\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

32) _____

    A) $x_1 = 5 - 2x_2 + 3x_3$ B) $x_1 = 15 + 11x_3$

           $x_2 = -5 - 4x_3$ $x_2 = -5 - 4x_3$

           x_3 is free $x_3 = 0$

    C) $x_1 = 5 - 2x_2 + 3x_3$ D) $x_1 = 15 + 11x_3$

           x_2 is free $x_2 = -5 - 4x_3$

           x_3 is free x_3 is free

33) $\begin{bmatrix} 1 & 0 & 6 & 3 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

33) _____

    A) $x_1 = 3 - 6x_3$ B) $x_1 = 3 - 6x_3$ C) No solution   D) $x_1 = 3 - 6x_3$

           x_2 is free $x_2 = -3 + 2x_3$ $x_2 = -3 + 2x_3$

           $x_3 = \dfrac{3}{2} + \dfrac{1}{2}x_2$ x_3 is free $x_3 = 0$

34) $\begin{bmatrix} 1 & 4 & -2 & -3 & 1 \\ 0 & 0 & 1 & 4 & -4 \\ -1 & -4 & 0 & -5 & 7 \end{bmatrix}$

A) $x_1 = -7 - 4x_2 - 5x_4$
   x_2 is free
   $x_3 = -4 - 4x_4$
   $x_4 = 0$

B) $x_1 = -4x_2 + 2x_3 + 3x_4 + 1$
   x_2 is free
   $x_3 = -4 - 4x_4$
   x_4 is free

C) $x_1 = -7 - 4x_2 - 5x_3$
   $x_2 = -4 - 4x_3$
   x_3 is free

D) $x_1 = -7 - 4x_2 - 5x_4$
   x_2 is free
   $x_3 = -4 - 4x_4$
   x_4 is free

35) $\begin{bmatrix} 1 & 5 & 8 & -1 & 2 & 5 \\ 0 & 0 & 0 & -4 & 3 & 4 \\ 0 & 0 & 0 & 0 & -2 & 8 \end{bmatrix}$

A) $x_1 = -5x_2 - 8x_3 + x_4 - 2x_5 + 5$
   x_2 is free
   x_3 is free
   $x_4 = \dfrac{3}{4} x_5 - 1$
   $x_5 = -4$

B) No solution

C) $x_1 = -5x_2 - 8x_3 + 9$
   x_2 is free
   x_3 is free
   $x_4 = -4$
   $x_5 = -4$

D) $x_1 = -5x_2 - 8x_3 + 9$
   x_2 is free
   $x_3 = -4$
   $x_4 = \dfrac{3}{4} x_5 - 1$
   $x_5 = -4$

Find the indicated vector.

36) Let $u = \begin{bmatrix} -9 \\ 7 \end{bmatrix}$, $v = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$. Find $u + v$.

A) $\begin{bmatrix} -6 \\ 3 \end{bmatrix}$ B) $\begin{bmatrix} -5 \\ 4 \end{bmatrix}$ C) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} -12 \\ 11 \end{bmatrix}$

37) Let $u = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$. Find $u - v$.

A) $\begin{bmatrix} -5 \\ 4 \end{bmatrix}$ B) $\begin{bmatrix} -2 \\ -7 \end{bmatrix}$ C) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ D) $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$

38) Let $u = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$, $v = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$. Find v - u. 38) _____

   A)   B)   C)   D)

$\begin{bmatrix} -6 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ $\begin{bmatrix} -7 \\ -15 \end{bmatrix}$ $\begin{bmatrix} -10 \\ -12 \end{bmatrix}$

39) Let $u = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$. Find 5u. 39) _____

   A)   B)   C)   D)

$\begin{bmatrix} -30 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 30 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 30 \\ -5 \end{bmatrix}$ $\begin{bmatrix} -30 \\ -5 \end{bmatrix}$

40) Let $u = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$. Find 7u. 40) _____

   A)   B)   C)   D)

$\begin{bmatrix} -28 \\ -49 \end{bmatrix}$ $\begin{bmatrix} 28 \\ -49 \end{bmatrix}$ $\begin{bmatrix} -28 \\ 49 \end{bmatrix}$ $\begin{bmatrix} 28 \\ 49 \end{bmatrix}$

41) Let $u = \begin{bmatrix} -9 \\ -2 \end{bmatrix}$. Find -3u. 41) _____

   A)   B)   C)   D)

$\begin{bmatrix} -27 \\ -6 \end{bmatrix}$ $\begin{bmatrix} -27 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 27 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 27 \\ -6 \end{bmatrix}$

42) Let $u = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$. Find -6u. 42) _____

   A)   B)   C)   D)

$\begin{bmatrix} -42 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 42 \\ 6 \end{bmatrix}$ $\begin{bmatrix} 42 \\ -6 \end{bmatrix}$ $\begin{bmatrix} -42 \\ -6 \end{bmatrix}$

43) Let $u = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$, $v = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$. Find -2u + 5v. 43) _____

   A)   B)   C)   D)

$\begin{bmatrix} 34 \\ -2 \end{bmatrix}$ $\begin{bmatrix} -8 \\ -65 \end{bmatrix}$ $\begin{bmatrix} -26 \\ 38 \end{bmatrix}$ $\begin{bmatrix} 22 \\ 10 \end{bmatrix}$
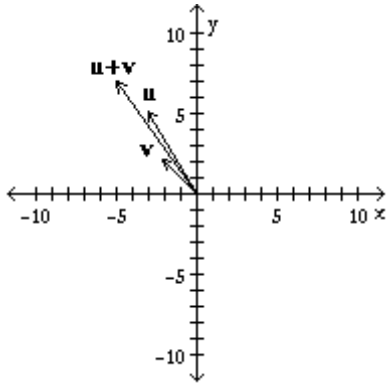
Display the indicated vector(s) on an xy-graph.

44) Let $u = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Display the vectors u, v, and u + v on the same axes.
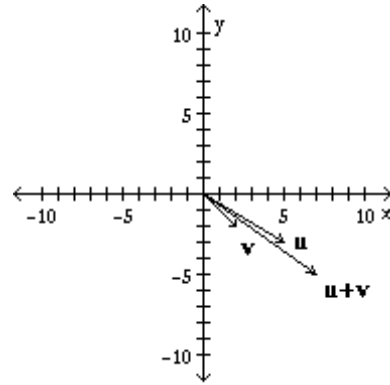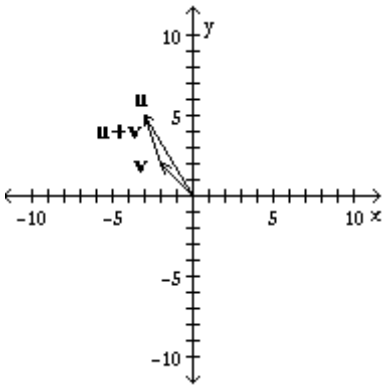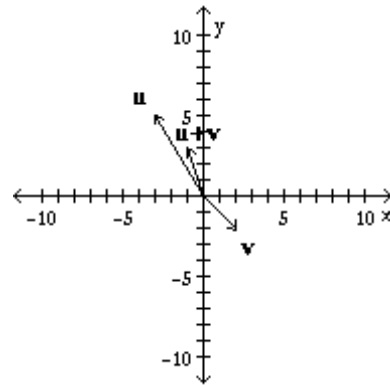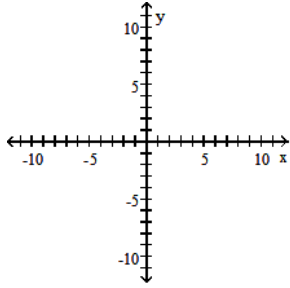
44) _____
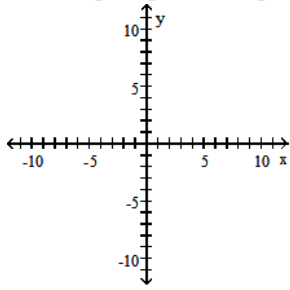
A)

B)

C)

D)
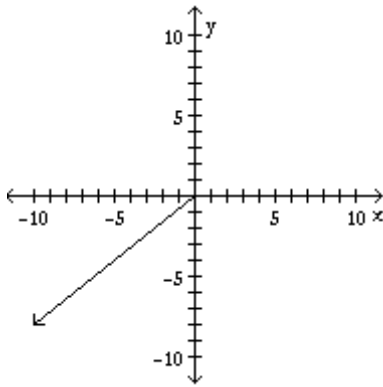
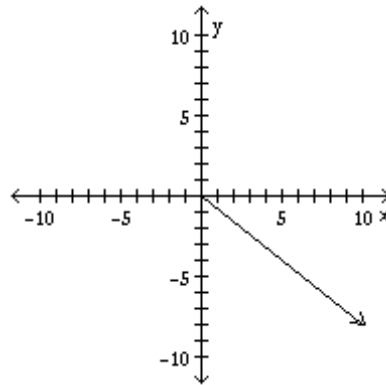45) Let u = $\begin{bmatrix} 5 \\ -4 \end{bmatrix}$ Display the vector 2u using the given axes.
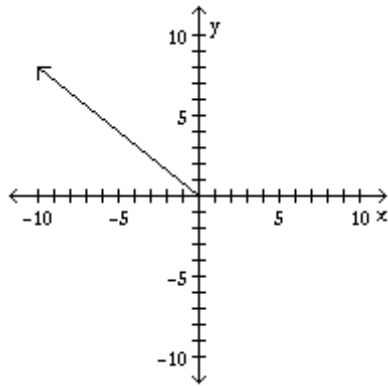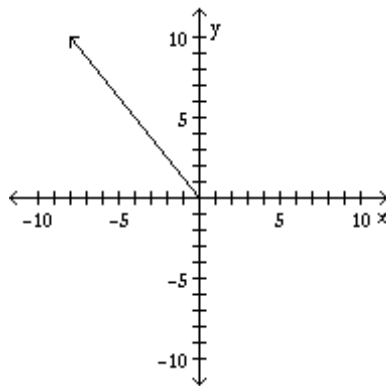
A)

B)

C)

D)

Solve the problem.

46) Let $a_1 = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$, $a_2 = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$, and $b = \begin{bmatrix} 2 \\ -10 \\ 6 \end{bmatrix}$.

Determine whether b can be written as a linear combination of a_1 and a_2. In other words, determine whether weights x_1 and x_2 exist, such that $x_1 a_1 + x_2 a_2 = b$. Determine the weights x_1 and x_2 if possible.

   A) No solution   B) $x_1 = -1$, $x_2 = -3$ C) $x_1 = -2$, $x_2 = -2$ D) $x_1 = -2$, $x_2 = -1$

47) Let $a_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $a_2 = \begin{bmatrix} -3 \\ -4 \\ 1 \end{bmatrix}$, $a_3 = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$, and $b = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.   47) _____

Determine whether b can be written as a linear combination of a_1, a_2, and a_3. In other words, determine whether weights x_1, x_2, and x_3 exist, such that $x_1 a_1 + x_2 a_2 + x_3 a_3 = b$. Determine the weights x_1, x_2, and x_3 if possible.

    A) $x_1 = -3$, $x_2 = 0$, $x_3 = 1$ B) $x_1 = -2$, $x_2 = -1$, $x_3 = 1$

    C) $x_1 = 2$, $x_2 = 1$, $x_3 = 0$ D) No solution

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

48) A company manufactures two products. For $1.00 worth of product A, the company   48) _____
spends $0.40 on materials, $0.25 on labor, and $0.10 on overhead. For $1.00 worth of product B, the company spends $0.50 on materials, $0.20 on labor, and $0.10 on overhead. Let

$$a = \begin{bmatrix} 0.40 \\ 0.25 \\ 0.10 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.10 \end{bmatrix}.$$

Then a and b represent the "costs per dollar of income" for the two products.
Evaluate 100a + 400b and give an economic interpretation of the result.

49) A company manufactures two products. For $1.00 worth of product A, the company   49) _____
spends $0.50 on materials, $0.20 on labor, and $0.15 on overhead. For $1.00 worth of product B, the company spends $0.45 on materials, $0.20 on labor, and $0.15 on overhead. Let

$$a = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.15 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0.45 \\ 0.20 \\ 0.15 \end{bmatrix}.$$

Then a and b represent the "costs per dollar of income" for the two products.
Suppose the company manufactures x_1 dollars worth of product A and x_2 dollars worth of product B and that its total costs for materials are $140, its total costs for labor are $60, and its total costs for overhead are $45.
Determine x_1 and x_2, the dollars worth of each product produced. Include a vector equation as part of your solution.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Compute the product or state that it is undefined.

50) $[-7 \ 2 \ 7] \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$ 50) _____

    A)   B)   C)   D)
     $[-21 \ 0 \ -21]$ $[-42]$ $[141]$ $\begin{bmatrix} -21 \\ 0 \\ -21 \end{bmatrix}$

51) $\begin{bmatrix} -2 & -2 & 6 \\ 5 & 8 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ -3 \\ 3 \end{bmatrix}$

51) _____

A) $\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 \\ 8 \end{bmatrix}$

C) $\begin{bmatrix} -2 & -2 & 6 \\ 5 & 8 & -5 \\ 8 & -3 & 3 \end{bmatrix}$

D) $[8 \ 1]$

52)

52) _____

$\begin{bmatrix} -1 & 3 \\ -8 & -5 \\ -6 & -8 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

A) Undefined

B) $\begin{bmatrix} -6 \\ -19 \\ -10 \end{bmatrix}$

C) $\begin{bmatrix} -3 & 8 \\ 9 & 5 \\ -6 & -8 \end{bmatrix}$

D) $\begin{bmatrix} -3 & -3 \\ -24 & 5 \\ -18 & 8 \end{bmatrix}$

53)

53) _____

$\begin{bmatrix} 5 & -3 \\ -3 & 4 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$

A) $\begin{bmatrix} 10 & -6 \\ 12 & -16 \\ -24 & 64 \end{bmatrix}$

B) $\begin{bmatrix} 4 \\ -4 \\ 40 \end{bmatrix}$

C) Undefined

D) $[-2 \ 42]$

Write the system as a vector equation or matrix equation as indicated.

54) Write the following system as a vector equation involving a linear combination of vectors.

54) _____

$6x_1 - 6x_2 - x_3 = 5$

$6x_1 + \quad 3x_3 = -5$

A) $x_1 \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$

B) $x_1 \begin{bmatrix} 6 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

C) $6 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - 6 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$

D) $x_1 \begin{bmatrix} 6 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

55) Write the following system as a matrix equation involving the product of a matrix and a vector on th side and a vector on the right side.

55) _____

$2x_1 + x_2 - 6x_3 = -6$

$6x_1 - 4x_2 \quad = 2$

A) $\begin{bmatrix} 2 & 1 & -6 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 1 & -6 \\ 6 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 6 \\ 1 & -4 \\ -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

D) $\begin{bmatrix} x_1 & x_2 & x_3 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

11

Solve the problem.

56) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -4 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.   56) _____

Determine if the equation $Ax = b$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible b_1, b_2, b_3, give a description of the set of all b for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

  A) Equation is consistent for all b_1, b_2, b_3 satisfying $2b_1 + b_2 = 0$.

  B) Equation is consistent for all possible b_1, b_2, b_3.

  C) Equation is consistent for all b_1, b_2, b_3 satisfying $-3b_1 + b_3 = 0$.

  D) Equation is consistent for all b_1, b_2, b_3 satisfying $7b_1 + 5b_2 + b_3 = 0$.

57) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -6 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.   57) _____

Determine if the equation $Ax = b$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible b_1, b_2, b_3, give a description of the set of all b for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

  A) Equation is consistent for all possible b_1, b_2, b_3.

  B) Equation is consistent for all b_1, b_2, b_3 satisfying $-3b_1 + b_3 = 0$.

  C) Equation is consistent for all b_1, b_2, b_3 satisfying $3b_1 + 3b_2 + b_3 = 0$.

  D) Equation is consistent for all b_1, b_2, b_3 satisfying $-b_1 + b_2 + b_3 = 0$.

58) Find the general solution of the simple homogeneous "system" below, which consists of a single linear equation. Give your answer as a linear combination of vectors. Let x_2 and x_3 be free variables   58) _____

$2x_1 - 16x_2 + 10x_3 = 0$

A)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

B)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 8 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 5 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

C)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -8 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

D)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 8 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

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59) Find the general solution of the homogeneous system below. Give your answer as a vector.   59) _____

$x_1 + 2x_2 - 3x_3 = 0$

$4x_1 + 7x_2 - 9x_3 = 0$

$-x_1 - 4x_2 + 9x_3 = 0$

A)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$$

C)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

D)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$$

60) Describe all solutions of $Ax = b$, where   60) _____

$$A = \begin{bmatrix} 2 & -5 & 3 \\ -2 & 6 & -5 \\ -4 & 7 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}.$$

Describe the general solution in parametric vector form.

A)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

C)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 7/2 \\ 2 \\ 0 \end{bmatrix}$$

D)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 7/2 \\ 2 \\ 1 \end{bmatrix}$$

61) Suppose an economy consists of three sectors: Energy (E), Manufacturing (M), and Agriculture (A).   61) _____
Sector E sells 70% of its output to M and 30% to A.
Sector M sells 30% of its output to E, 50% to A, and retains the rest.
Sector A sells 15% of its output to E, 30% to M, and retains the rest.

Denote the prices (dollar values) of the total annual outputs of the Energy, Manufacturing, and Agri
sectors by p_e, p_m, and p_a, respectively. If possible, find equilibrium prices that make each sector's in
match its expenditures.
Find the general solution as a vector, with p_a free.

A)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.308\, p_a \\ 0.716\, p_a \\ p_a \end{bmatrix}$$

B)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.607\, p_a \\ 0.481\, p_a \\ p_a \end{bmatrix}$$

C)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.356\, p_a \\ 0.686\, p_a \\ p_a \end{bmatrix}$$

D)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.465\, p_a \\ 0.593\, p_a \\ p_a \end{bmatrix}$$

62) The network in the figure shows the traffic flow (in vehicles per hour) over several one-way streets   62) _____
in the downtown area of a certain city during a typical lunch time. Determine the general flow patte
the network.
In other words, find the general solution of the system of equations that describes the flow. In your
general solution let x_4 be free.
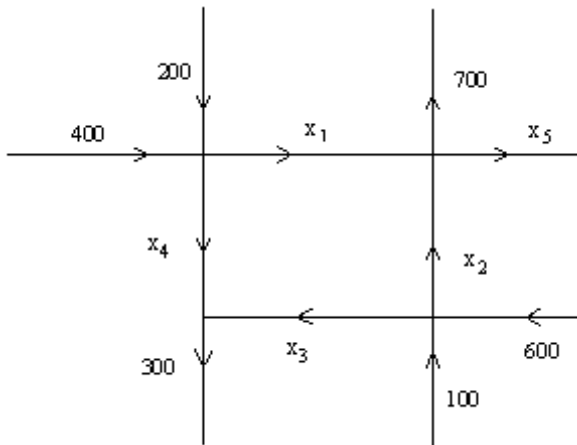
A) $x_1 = 600 - x_4$
   $x_2 = 400 + x_4$
   $x_3 = 300 - x_4$
   x_4 is free
   $x_5 = 300$

B) $x_1 = 500 + x_4$
   $x_2 = 400 - x_4$
   $x_3 = 300 - x_4$
   x_4 is free
   $x_5 = 200$

C) $x_1 = 600 + x_5$
   $x_2 = 400 - x_5$
   $x_3 = 300 - x_5$
   $x_4 = 300$
   x_5 is free

D) $x_1 = 600 - x_4$
   $x_2 = 400 - x_4$
   $x_3 = 300 + x_4$
   x_4 is free
   $x_5 = 300$

63) Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 8 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ -2 \\ -6 \end{bmatrix}$.   63) _____

Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent.
  A) No   B) Yes

64) Determine if the columns of the matrix $A = \begin{bmatrix} -2 & 1 & 4 \\ 4 & 0 & -4 \\ 2 & 1 & 0 \end{bmatrix}$ are linearly independent.   64) _____

  A) Yes   B) No

65) For what values of h are the given vectors linearly independent?   65) _____
$\begin{bmatrix} -5 \\ -6 \\ 5 \end{bmatrix}, \begin{bmatrix} 20 \\ 24 \\ h \end{bmatrix}$
  A) Vectors are linearly independent for all h
  B) Vectors are linearly dependent for all h
  C) Vectors are linearly independent for $h \neq -20$
  D) Vectors are linearly independent for $h = -20$

66) For what values of h are the given vectors linearly dependent?   66) _____

$$\begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ h \end{bmatrix}$$

A) Vectors are linearly dependent for h = -2   B) Vectors are linearly dependent for h ≠ -2
C) Vectors are linearly independent for all h   D) Vectors are linearly dependent for all h

67) Let A = $\begin{bmatrix} 2 & 3 & 1 \\ 8 & -7 & 5 \end{bmatrix}$ and u = $\begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}$.   67) _____

Define a transformation T: R^3 -> R^2 by T(x) = Ax. Find T(u), the image of u under the transformation T.

A)   B)   C)   D)

$\begin{bmatrix} 18 \\ 42 \end{bmatrix}$ $\begin{bmatrix} 6 & 21 & 2 \\ 24 & -49 & 10 \end{bmatrix}$ $\begin{bmatrix} 30 \\ -28 \\ 12 \end{bmatrix}$ $\begin{bmatrix} 29 \\ -15 \end{bmatrix}$

68) Let T: R^2 -> R^2 be a linear transformation that maps u = $\begin{bmatrix} -6 \\ 4 \end{bmatrix}$ into $\begin{bmatrix} -22 \\ 12 \end{bmatrix}$ and maps v = $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ into   68) _____

$\begin{bmatrix} 11 \\ -4 \end{bmatrix}$.

Use the fact that T is linear to find the image of 3u + v.

A)   B)   C)   D)

$\begin{bmatrix} -33 \\ 24 \end{bmatrix}$ $\begin{bmatrix} -55 \\ 32 \end{bmatrix}$ $\begin{bmatrix} -11 \\ 8 \end{bmatrix}$ $\begin{bmatrix} -16 \\ 7 \end{bmatrix}$

69) Let A = $\begin{bmatrix} 1 & -3 & 0 \\ -4 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}$ and b = $\begin{bmatrix} 9 \\ -6 \\ 4 \end{bmatrix}$.   69) _____

Define a transformation T: R^3 -> R^3 by T(x) = Ax.
If possible, find a vector x whose image under T is b. Otherwise, state that b is not in the range of the transformation T.

A)
$\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$

B)
b is not in the range of the transformation T.

C)
$\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$

D)
$\begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$

70) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 4 & -1 \\ 2 & -5 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 4 \\ -4 \end{bmatrix}$.

70) _____

Define a transformation T: $\mathcal{R}^3 \to \mathcal{R}^3$ by T(x) = Ax.
If possible, find a vector x whose image under T is b. Otherwise, state that b is not in the range of the transformation T.

A)

$\begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$

B)

$\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$

C)

b is not in the range of the transformation T.

D)

$\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$

Describe geometrically the effect of the transformation T.

71) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

71) _____

Define a transformation T by T(x) = Ax.

A) Horizontal shear   B) Vertical shear

C) Projection onto x_1-axis   D) Projection onto x_2-axis

72) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

72) _____

Define a transformation T by T(x) = Ax.

A) Projection onto the x_2-axis   B) Projection onto the $x_2 x_3$-plane

C) Vertical shear   D) Horizontal shear

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Solve the problem.

73) The columns of $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. 73) _____

Suppose that T is a linear transformation from $\mathcal{R}^3$ into $\mathcal{R}^2$ such that

$$T(e_1) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, T(e_2) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \text{ and } T(e_3) = \begin{bmatrix} -5 \\ 1 \end{bmatrix}.$$

Find a formula for the image of an arbitrary $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathcal{R}^3$.

A)
$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_2 \\ 5x_1 \\ 5x_2 + x_3 \end{bmatrix}$$

B)
$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 5x_2 - 5x_3 \\ -2x_1 \qquad + x_3 \end{bmatrix}$$

C)
$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 5x_2 - 5x_3 \\ 5x_1 \\ -2x_1 + x_3 \end{bmatrix}$$

D)
$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_2 \\ 5x_1 \end{bmatrix}$$

Find the standard matrix of the linear transformation T.

74) $T: \mathcal{R}^2 \to \mathcal{R}^2$ rotates points (about the origin) through $\dfrac{7}{4}\pi$ radians (with counterclockwise rotation for a positive angle). 74) _____

A)
$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

B)
$$\begin{bmatrix} \dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \\ -\dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \end{bmatrix}$$

C)
$$\begin{bmatrix} -\dfrac{\sqrt{2}}{2} & -\dfrac{\sqrt{2}}{2} \\ -\dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} \end{bmatrix}$$

D)
$$\begin{bmatrix} \dfrac{\sqrt{3}}{3} & \dfrac{\sqrt{3}}{3} \\ -\dfrac{\sqrt{3}}{3} & \dfrac{\sqrt{3}}{3} \end{bmatrix}$$

75) $T: \mathcal{R}^2 \to \mathcal{R}^2$ first performs a vertical shear that maps e_1 into $e_1 + 3e_2$, but leaves the vector e_2 unchanged, then reflects the result through the horizontal x_1-axis. 75) _____

A)
$$\begin{bmatrix} -1 & 0 \\ 3 & -1 \end{bmatrix}$$

B)
$$\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$$

C)
$$\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$$

D)
$$\begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix}$$

Determine whether the linear transformation T is one-to-one and whether it maps as specified.

76) Let T be the linear transformation whose standard matrix is 76) _____

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -4 \\ -2 & -9 & 5 \end{bmatrix}.$$

Determine whether the linear transformation T is one-to-one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^3$.

A) Not one-to-one; not onto $\mathcal{R}^3$

B) One-to-one; onto $\mathcal{R}^3$

C) Not one-to-one; onto $\mathcal{R}^3$

D) One-to-one; not onto $\mathcal{R}^3$

77) $T(x_1, x_2, x_3) = (-4x_2 - 4x_3, -2x_1 + 8x_2 + 4x_3, -x_1 - 2x_3, 4x_2 + 4x_3)$ 77) _____

   Determine whether the linear transformation T is one-to-one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^4$.

   A) One-to-one; not onto $\mathcal{R}^4$ B) One-to-one; onto $\mathcal{R}^4$
   C) Not one-to-one; not onto $\mathcal{R}^4$ D) Not one-to-one; onto $\mathcal{R}^4$

Solve the problem.

78) The table shows the amount (in g) of protein, carbohydrate, and fat supplied by one unit (100 g) of th   78) _____
   different foods.

	Food 1	Food 2	Food 3
Protein	15	35	25
Carbohydrate	45	30	50
Fat	6	4	1

   Betty would like to prepare a meal using some combination of these three foods. She would like the
   to contain 15 g of protein, 25 g of carbohydrate, and 3 g of fat. How many units of each food should
   use so that the meal will contain the desired amounts of protein, carbohydrate, and fat? Round to 3
   decimal places.
      A) 0.360 units of Food 1, 0.204 units of Food 2, 0.055 units of Food 3
      B) 0.326 units of Food 1, 0.247 units of Food 2, 0.059 units of Food 3
      C) 0.302 units of Food 1, 0.238 units of Food 2, 0.085 units of Food 3
      D) 0.280 units of Food 1, 0.192 units of Food 2, 0.164 units of Food 3

79) The population of a city in 2000 was 400,000 while the population of the suburbs of that city in 2000   79) _____
   was 900,000.
   Suppose that demographic studies show that each year about 5% of the city's population moves to th
   suburbs (and 95% stays in the city), while 4% of the suburban population moves to the city (and
   96% remains in the suburbs).
   Compute the population of the city and of the suburbs in the year 2002. For simplicity, ignore other
   influences on the population such as births, deaths, and migration into and out of the city/suburban
      A) City: 430,560   B) City: 416,000
         Suburbs: 869,440   Suburbs: 884,000
      C) City: 361,000   D) City: 361,000
         Suburbs: 939,000   Suburbs: 829,440

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1) C
2) B
3) D
4) A
5) C
6) D
7) C
8) C
9) D
10) B
11) B
12) A
13) A
14) A
15) A
16) B
17) B
18) B
19) B
20) A
21) C
22) B
23) C
24) C
25) C
26) B
27) B
28) C
29) C
30) B
31) D
32) D
33) B
34) D
35) C
36) D
37) C
38) D
39) B
40) B
41) C
42) A
43) A
44) A
45) B
46) C
47) D

48) $100a + 400b = \begin{bmatrix} 240 \\ 105 \\ 50 \end{bmatrix}$

$100a + 400b$ lists the various costs for producing $100 worth of product A and $400 worth of product B, namely $240 for materials, $105 for labor, and $50 for overhead.

49) $x_1 a + x_2 b = \begin{bmatrix} 140 \\ 60 \\ 45 \end{bmatrix}$

or

$x_1 \begin{bmatrix} 0.50 \\ 0.20 \\ 0.15 \end{bmatrix} + x_2 \begin{bmatrix} 0.45 \\ 0.20 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 140 \\ 60 \\ 45 \end{bmatrix}$

$x_1 = 100, x_2 = 200$

50) B
51) A
52) B
53) C
54) B
55) A
56) B
57) C
58) A
59) C
60) D
61) C
62) A
63) B
64) B
65) C
66) D
67) D
68) B
69) C
70) C
71) B
72) B
73) B
74) B
75) D
76) B
77) C
78) B
79) A