

Instructor's Manual

Mathematics for Economics and Business

Seventh edition

Ian Jacques

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SOLUTIONS TO PROBLEMS

CHAPTER 1

Linear Equations

Section 1.1 Introduction to algebra

Practice Problems

- (a) -30 (b) 2 (c) -5
(d) 5 (e) 36 (f) -1
- (a) -1 (b) -7 (c) 5
(d) 0 (e) -91 (f) -5
- (a) 19 (b) 1500 (c) 32
(d) 35
- (a) $x + 9y$ (b) $2y + 4z$
(c) not possible
(d) $8r^2 + s + rs - 3s^2$ (e) $-4f$
(f) not possible (g) 0
- (a) $5z - 2z^2$
(b) $6x - 6y + 3y - 6x = -3y$
(c) $x - y + z - x^2 - x + y = z - x^2$
- (a) $7(d + 3)$ (b) $4(4w - 5q)$
(c) $3(3x - y + 3z)$ (d) $5Q(1 - 2Q)$
- (a) $x^2 - 2x + 3x - 6 = x^2 + x - 6$
(b) $x^2 - xy + yx - y^2 = x^2 - y^2$
(c) $x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$

$$\begin{aligned} \text{(d)} \quad & 5x^2 - 5xy + 5x + 2yx - 2y^2 + 2y \\ & = 5x^2 - 3xy + 5x - 2y^2 + 2y \end{aligned}$$

8. (a) $(x + 8)(x - 8)$

(b) $(2x + 9)(2x - 9)$

Exercise 1.1 (p. 18)

1. (a) -20 (b) 3 (c) -4 (d) 1

(e) -12 (f) 5 (g) -5 (h) 3

(i) 30 (j) 4.

2. (a) -1 (b) -3 (c) -11 (d) 16

(e) -1 (f) -13 (g) 11 (h) 0

(i) -31 (j) -2

3. (a) -3 (b) 2 (c) 18 (d) -15

(e) -41 (f) -3 (g) 18 (h) -6

(i) -25 (j) -6.

4. (a) $2PQ$ (b) $8I$ (c) $3xy$ (d) $4qwz$

(e) b^2 (f) $3k^2$

5. (a) $19w$ (b) $4x - 7y$ (c) $9a + 2b - 2c$

(d) $x^2 + 2x$ (e) $4c - 3cd$ (f) $2st + s^2 + t^2 + 9$.

6. (a) 10 (b) 18 (c) 2000

(d) 96 (e) 70

7. (a) 1 (b) 5 (c) -6 (d) -6

(e) -30 (f) 44.

8. (a) 16.

(b) Presented with the calculation, -4^2 , your calculator uses BIDMAS, so squares first to get 16 and then subtracts from zero to give a final answer, -16. To obtain the correct answer you need to use brackets:

$$\boxed{(-4)^2} \quad \boxed{-4^2} \quad \boxed{4^2} \quad \boxed{-4^2} \quad \boxed{-4^2}$$

9. (a) 9 (b) 21 no.
10. (a) 43.96 (b) 1.13 (c) 10.34 (d) 0.17
 (e) 27.38 (f) 3.72 (g) 62.70 (h) 2.39
11. (a) $7x - 7y$ (b) $15x - 6y$ (c) $4x + 12$ (d) $21x - 7$
 (e) $3x + 3y + 3z$ (f) $3x^2 - 4x$ (g) $y + 2z - 2x - 6y + 2z = -2x - 5y + 4z$
12. (a) $5(5c + 6)$ (b) $9(x - 2)$ (c) $x(x + 2)$ (d) $4(4x - 3y)$
 (e) $2x(2x - 3y)$ (f) $5(2d - 3e + 10)$
13. (a) $x^2 + 7x + 10$ (b) $a^2 + 3a - 4$ (c) $d^2 - 5d - 24$
 (d) $6s^2 + 23s + 21$ (e) $2y^2 + 5y + 3$ (f) $10t^2 - 11t - 14$
 (g) $9n^2 - 4$ (h) $a^2 - 2ab + b^2$
14. (a) $6x + 2y$ (b) $11x^2 - 3x - 3$ (c) $14xy + 2x$
 (d) $6xyz + 2xy$ (e) $10a - 2b$ (f) $17x + 22y$
 (g) $11 - 3p$ (h) $x^2 + 10x$
15. (a) $(x + 2)(x - 2)$ (b) $(Q + 7)(Q - 7)$ (c) $(x + y)(x - y)$
 (d) $(3x + 10y)(3x - 10y)$
16. (a) $4x^2 + 8x - 2$ (b) $3x^2 + 2x - 3x^2 - 15x = -13x$

Section 1.2 Further algebra

Practice Problems

1. (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2y}$ (d) $\frac{1}{2+3x}$ (e) $\frac{1}{x-4}$
2. (a) $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$
 (b) $17 \times \frac{1}{14_2} = \frac{1}{2}$

$$(c) \frac{2}{3} \div \frac{8}{9} = \frac{\cancel{2}^1}{3} \times \frac{9^3}{\cancel{8}_4} = \frac{3}{4}$$

$$(d) \frac{8}{9} \div 16 = \frac{\cancel{8}^1}{9} \times \frac{1}{\cancel{16}_2} = \frac{1}{18}$$

$$3. (a) \frac{3}{7} - \frac{1}{7} = \frac{2}{7}$$

$$(b) \frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

$$(c) \frac{7}{18} - \frac{1}{4} = \frac{14}{36} - \frac{9}{36} = \frac{5}{36}$$

$$4. (a) \frac{5}{\cancel{x-1}} \times \frac{\cancel{x-1}}{x+2} = \frac{5}{x+2}$$

$$(b) \frac{x^2}{x+10} \div \frac{x}{x+1} = \frac{x^2}{x+10} \times \frac{x+1}{x} = \frac{x(x+1)}{x+10}$$

$$(c) \frac{4}{x+1} + \frac{1}{x+1} = \frac{4+1}{x+1} = \frac{5}{x+1}$$

$$(d) \frac{2}{x+1} - \frac{1}{x+2}$$

$$= \frac{2(x+2)}{(x+1)(x+2)} - \frac{(1)(x+1)}{(x+1)(x+2)}$$

$$= \frac{(2x+4) - (x+1)}{(x+1)(x+2)} = \frac{(x+3)}{(x+1)(x+2)}$$

$$5. (a) 4x + 1 = 25$$

$$4x = 24$$

(subtract 1 from both sides)

$$x = 6$$

(divide both sides by 4)

$$(b) 4x + 5 = 5x - 7$$

$$5 = x - 7$$

(subtract 4x from both sides)

$$12 = x$$

(add 7 to both sides)

(c) $3(3 - 2x) + 2(x - 1) = 10$

$$9 - 6x + 2x - 2 = 10 \quad (\text{multiply out brackets})$$

$$7 - 4x = 10 \quad (\text{collect like terms})$$

$$-4x = 3 \quad (\text{subtract 7 from both sides})$$

$$x = -\frac{3}{4} \quad (\text{divide both sides by } -4)$$

(d) $\frac{4}{x-1} = 5$

$$4 = 5(x-1) \quad (\text{multiply both sides by } x-1)$$

$$4 = 5x - 5 \quad (\text{multiply out brackets})$$

$$9 = 5x \quad (\text{add 5 to both sides})$$

$$\frac{9}{5} = x \quad (\text{divide both sides by 5})$$

(e) $\frac{3}{x} = \frac{5}{x-1}$

$$3(x-1) = 5x \quad (\text{cross-multiplication})$$

$$3x - 3 = 5x \quad (\text{multiply out brackets})$$

$$-3 = 2x \quad (\text{subtract } 3x \text{ from both sides})$$

$$-\frac{3}{2} = x \quad (\text{divide both sides by 2})$$

6. (a) $12 > 9$ (true) (b) $12 > 6$ (true)

(c) $3 > 0$ (true) (d) same as (c)

(e) $2 > 1$ (true) (f) $-24 > -12$ (false)

(g) $-6 > -3$ (false) (h) $2 > -1$ (false)

(i) $-4 > -7$ (true).

7. (a) $2x < 3x + 7$

$$-x < 7 \quad (\text{subtract } 3x \text{ from both sides})$$

$$x > -7 \quad (\text{divide both sides by } -1 \text{ changing sense because } -1 < 0)$$

(b) $21x - 19 \geq 4x + 15$

$$17x - 19 \geq 15 \quad (\text{subtract } 4x \text{ from both sides})$$

$$17x \geq 34 \quad (\text{add 19 to both sides})$$

$$x \geq 2 \quad (\text{divide both sides by 17, leaving inequality unchanged because } 17 > 0)$$

Exercise 1.2 (p. 35)

1. (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{1}{3}$ (e) $\frac{4}{3} = 1\frac{1}{3}$
2. (a) $\frac{2x}{3}$ (b) $\frac{1}{2x}$ (c) $\frac{1}{ac}$ (d) $\frac{2}{3xy}$ (e) $\frac{3a}{4b}$
3. (a) $\frac{2p}{2(2q+3r)} = \frac{p}{2q+3r}$ (b) $\frac{x}{x(x-4)} = \frac{1}{x-4}$ (c) $\frac{3ab}{3a(2a+1)} = \frac{b}{2a+1}$
- (d) $\frac{14d}{7d(3-e)} = \frac{2}{3-e}$ (e) $\frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$ (using the difference of two squares for the denominator)
4. $\frac{x-1}{2x-2} = \frac{x-1}{2(x-1)} = \frac{1}{2}$; other two have no common factors on top and bottom.
5. (a) $\frac{3}{7}$ (b) $-\frac{1}{3}$ (c) $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$
- (d) $\frac{15}{20} - \frac{8}{20} = \frac{7}{20}$ (e) $\frac{3}{18} + \frac{4}{18} = \frac{7}{18}$ (f) $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$
- (g) $\frac{5}{2} \times \frac{3^1}{4} = \frac{5}{8}$ (h) $\frac{2^4}{5} \times \frac{3^1}{2} = \frac{2}{5}$ (i) $\frac{7}{4} \times \frac{2^1}{3} = \frac{7}{12}$
- (j) $\frac{1^2}{15} \times \frac{5^1}{4} = \frac{1}{30}$ (k) $\frac{2}{9} \times \frac{1}{3} = \frac{2}{27}$ (l) $\frac{3}{1} \times \frac{7}{2} = \frac{21}{2} = 10\frac{1}{2}$
6. (a) $\frac{2}{3x} + \frac{1}{3x} = \frac{3}{3x} = \frac{1}{x}$ (b) $\frac{2}{1x} \times \frac{x^1}{5} = \frac{2}{5}$ (c) $\frac{3}{x} - \frac{2}{x^2} = \frac{3x}{x^2} - \frac{2}{x^2} = \frac{3x-2}{x^2}$
- (d) $\frac{7}{x} + \frac{2}{y} = \frac{7y}{xy} + \frac{2x}{xy} = \frac{7y+2x}{xy}$ (e) $\frac{a}{2} \div \frac{a}{6} = \frac{1a}{2} \times \frac{6^3}{a_1} = 3$
- (f) $\frac{5c}{12} + \frac{5d}{18} = \frac{15c}{36} + \frac{10d}{36} = \frac{15c+10d}{36}$ (g) $\frac{x+2}{\cancel{1}y-5} \times \frac{\cancel{y-5}^1}{x+3} = \frac{x+2}{x+3}$
- (h) $\frac{4gh}{7} \div \frac{2g}{9h} = \frac{2^4gh}{7} \times \frac{9h}{2_1g} = \frac{18h^2}{7}$ (i) $\frac{t}{4} \div 5 = \frac{t}{4} \times \frac{1}{5} = \frac{t}{20}$ (j) $\frac{1P}{1Q} \times \frac{Q^1}{P_1} = 1$
7. (a) $x+2=7$
 $x=5$ (subtract 2 from both sides)

(b) $3x = 18$
 $x = 6$ (divide both sides by 3)

(c) $\frac{x}{9} = 2$
 $x = 18$ (multiply both sides by 9)

(d) $x - 4 = -2$
 $x = 2$ (add 4 to both sides)

(e) $2x - 3 = 17$
 $2x = 20$ (add 3 to both sides)
 $x = 10$ (divide both sides by 2)

(f) $3x + 4 = 1$
 $3x = -3$ (subtract 4 from both sides)
 $x = -1$ (divide both sides by 3)

(g) $\frac{x}{6} - 7 = 3$
 $\frac{x}{6} = 10$ (add 7 to both sides)
 $x = 60$ (multiply both sides by 6)

(h) $3(x - 1) = 2$
 $3x - 3 = 2$ (multiply out brackets)
 $3x = 5$ (add 3 to both sides)
 $x = \frac{5}{3} = 1\frac{2}{3}$ (divide both sides by 3)

(i) $4 - x = 9$
 $-x = 5$ (subtract 4 from both sides)
 $x = -5$ (divide both sides by -1)

(j) $6x + 2 = 5x - 1$
 $x + 2 = -1$ (subtract $5x$ from both sides)
 $x = -3$ (subtract 2 from both sides)

(k) $5(3x + 8) = 10$
 $15x + 40 = 10$ (multiply out brackets)
 $15x = -30$ (subtract 40 from both sides)
 $x = -2$ (divide both sides by 15)

(l) $2(x - 3) = 5(x + 1)$

$2x - 6 = 5x + 5$ (multiply out brackets)

$-3x - 6 = 5$ (subtract $5x$ from both sides)

$-3x = 11$ (add 6 to both sides)

$x = \frac{-11}{3} = -3\frac{2}{3}$ (divide both sides by -3)

(m) $\frac{4x - 7}{3} = 2$

$4x - 7 = 6$ (multiply both sides by 3)

$4x = 13$ (add 7 to both sides)

$x = \frac{13}{4} = 3\frac{1}{4}$ (divide both sides by 4)

(n) $\frac{4}{x+1} = 1$

$4 = x + 1$ (multiply both sides by $x + 1$)

$3 = x$ (subtract 1 from both sides)

(o) $5 - \frac{1}{x} = 1$

$5 = 1 + \frac{1}{x}$ (add $\frac{1}{x}$ to both sides)

$4 = \frac{1}{x}$ (subtract 1 from both sides)

$4x = 1$ (multiply both sides by x)

$x = \frac{1}{4}$ (divide both sides by 4)

8. (a), (d), (e), (f).

9. (a) $2x > x + 1$

$x > 1$ (subtract x from both sides)

(b) $7x + 3 \leq 9 + 5x$

$2x + 3 \leq 9$ (subtract $5x$ from both sides)

$2x \leq 6$ (subtract 3 from both sides)

$x \leq 3$ (divide both sides by 2)

(c) $x - 5 > 4x + 4$

$-3x - 5 > 4$ (subtract $4x$ from both sides)

$-3x > 9$ (add 5 to both sides)

$x < -3$ (divide both sides by -3)

(d) $x - 1 < 2x - 3$

$-x - 1 < -3$ (subtract $2x$ from both sides)

$-x < -2$ (add 1 to both sides)

$x > 2$ (divide both sides by -1)

10. $\frac{4}{x^2 y} \div \frac{2x}{y} = \frac{2 \cdot 4}{x^2 y} \times \frac{y}{2 \cdot x} = \frac{2}{x^3}$

11. (a) $6(2 + x) = 5(1 - 4x)$

$12 + 6x = 5 - 20x$ (multiply out brackets)

$12 + 26x = 5$ (add $20x$ to both sides)

$26x = -7$ (subtract 12 from both sides)

$x = -\frac{7}{26}$ (divide both sides by 26)

(b) $3x + 6 \geq 5x - 14$

$-2x + 6 \geq -14$ (subtract $5x$ from both sides)

$-2x \geq -20$ (subtract 6 from both sides)

$x \leq 10$ (divide both sides by -2)

Section 1.3 Graphs of linear equations

Practice Problems

1. From Figure S1.1 note that all five points lie on a straight line.

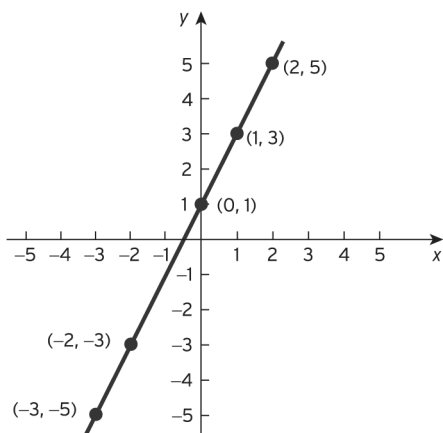


Figure S1.1

2.

Point	Check	
$(-1, 2)$	$2(-1) + 3(2) = -2 + 6 = 4$	✓
$(-4, 4)$	$2(-4) + 3(4) = -8 + 12 = 4$	✓
$(5, -2)$	$2(5) + 3(-2) = 10 - 6 = 4$	✓
$(2, 0)$	$2(2) + 3(0) = 4 + 0 = 4$	✓

The graph is sketched in Figure S1.2.

The graph shows that $(3, -1)$ does not lie on the line. This can be verified algebraically:

$$2(3) + 3(-1) = 6 - 3 = 3 \neq 4$$

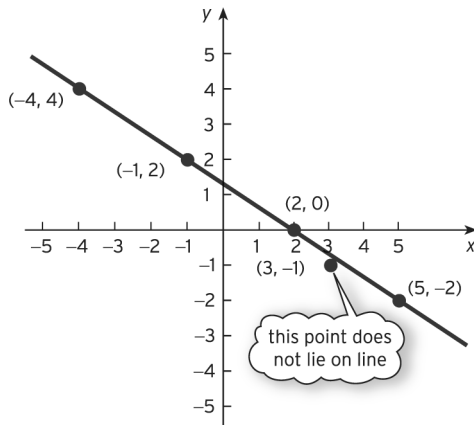


Figure S1.2

3. $3x - 2y = 4$
 $3(2) - 2y = 4$ (substitute $x = 2$)
 $6 - 2y = 4$
 $-2y = -2$ (subtract 6 from both sides)
 $y = 1$ (divide both sides by -2)

Hence $(2, 1)$ lies on the line.

$$3x - 2y = 4$$

$$3(-2) - 2y = 4$$

$$-6 - 2y = 4$$
 (substitute $x = -2$)
$$-2y = 10$$
 (add 6 to both sides)
$$y = -5$$
 (divide both sides by -2)

Hence $(-2, -5)$ lies on the line.

The line is sketched in Figure S1.3.

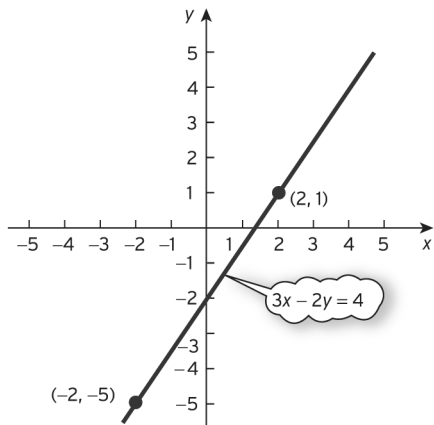


Figure S1.3

4. $x - 2y = 2$

$$0 - 2y = 2 \quad (\text{substitute } x = 0)$$

$$-2y = 2$$

$$y = -1 \quad (\text{divide both sides by } -2)$$

Hence $(0, -1)$ lies on the line.

$$x - 2y = 2$$

$$x - 2(0) = 2 \quad (\text{substitute } y = 0)$$

$$x - 0 = 2$$

$$x = 2$$

Hence $(2, 0)$ lies on the line.

The graph is sketched in Figure S1.4.

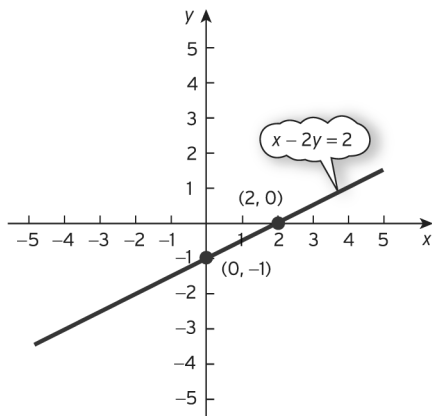


Figure S1.4

5. From Figure S1.5 the point of intersection is $(1, -\frac{1}{2})$.

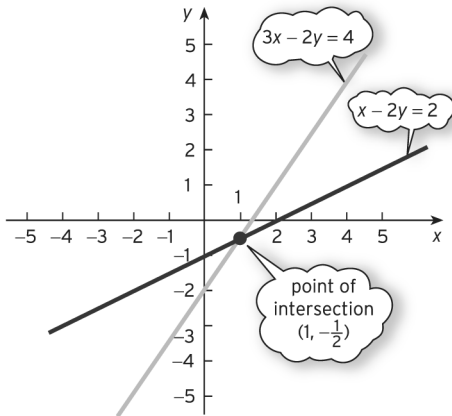


Figure S1.5

6. (a) $a = 1, b = 2$. The graph is sketched in Figure S1.6.

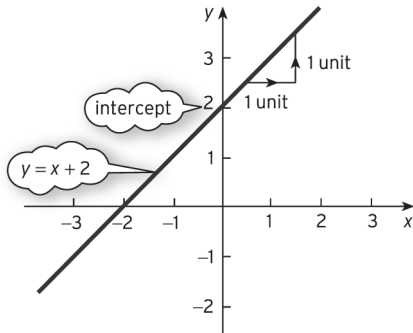


Figure S1.6

- (b) $4x + 2y = 1$

$$2y = 1 - 4x \quad (\text{subtract } 4x \text{ from both sides})$$

$$y = \frac{1}{2} - 2x \quad (\text{divide both sides by } 2)$$

so $a = -2, b = \frac{1}{2}$. The graph is sketched in Figure S1.7.

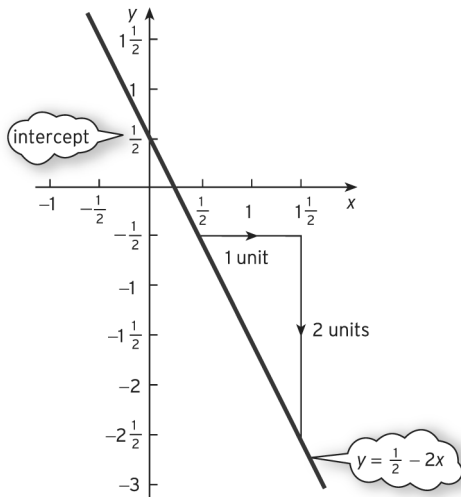


Figure S1.7

Exercise 1.3 (p. 49)

- From Figure S1.8 the point of intersection is (2, 3).

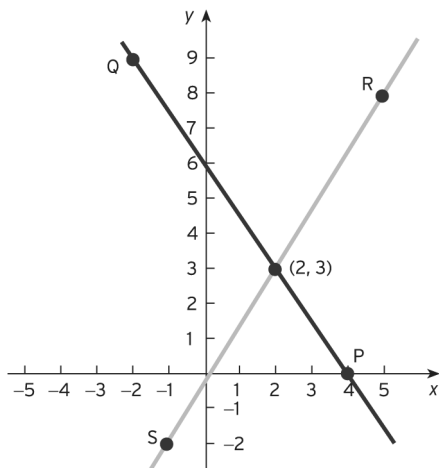


Figure S1.8

- A, C, D, E
- (a) $3x - 10 = 8$
 $3x = 18$ (add 10 to both sides)
 $x = 6$ (divide both sides by 3)

(b) $3 - 5y = 8$

$-5y = 5$ (subtract 3 from both sides)

$y = -1$ (divide both sides by -5)

$(6, 2), (1, -1)$.

4. $x \quad y$

0 8

6 0

3 4

The graph is sketched in Figure S1.9.

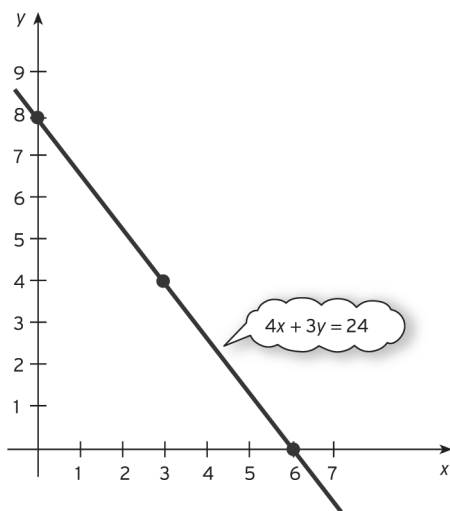


Figure S1.9

5. (a) The line $-2x + y = 2$ passes through $(0, 2)$ and $(-1, 0)$
The line $2x + y = -6$ passes through $(0, -6)$ and $(-3, 0)$
The point of intersection has coordinates $(-2, -2)$
- (b) The line $3x + 4y = 12$ passes through $(0, 3)$ and $(4, 0)$
The line $x + 4y = 8$ passes through $(0, 2)$ and $(8, 0)$
The point of intersection has coordinates $(2, 1\frac{1}{2})$

(c) The line $2x + y = 4$ passes through (0,4) and (2,0)

The line $4x - 3y = 3$ passes through (0,-1) and (3/4,0)

The point of intersection has coordinates $(1\frac{1}{2}, 1)$

(d) The line $x + y = 1$ passes through (0,1) and (1,0)

The line $6x + 5y = 15$ passes through (0,3) and (5/2,0)

The point of intersection has coordinates (10,-9)

6. (a) 5, 9 (b) 3, -1 (c) -1, 13

(d) $-x + y = 4$

$y = x + 4$ (add x to both sides)

so the slope is 1 and the y intercept is 4

(e) $4x + 2y = 5$

$2y = -4x + 5$ (subtract $4x$ from both sides)

$y = -2x + \frac{5}{2}$ (divide both sides by 2)

so the slope is -2 and the y intercept is $\frac{5}{2}$

(f) $5x - y = 6$

$-y = -5x + 6$ (subtract $5x$ from both sides)

$y = 5x - 6$ (divide both sides by -1)

so the slope is 5 and the y -intercept is -6

7. (a) The line has a slope of -1 and a y -intercept of 0.

The line passes through the origin and for every one unit along the graph goes down 1

The graph is sketched in Figure S1.10.

(b) $x - 2y = 6$

$-2y = -x + 6$ (subtract x from both sides)

$y = \frac{1}{2}x - 3$

so the line has a slope of $1/2$ and a y intercept of -3

The line passes through the point (0,-3) and for every unit along the graph goes up by $1/2$ unit, or equivalently, for every 2 units along it goes up by 1 unit.

The graph is sketched in Figure S1.11.

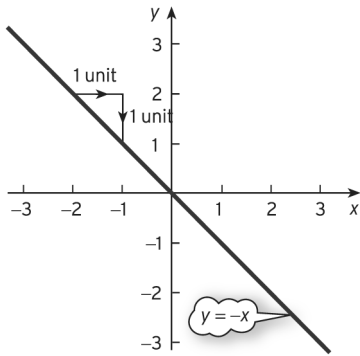


Figure S1.10

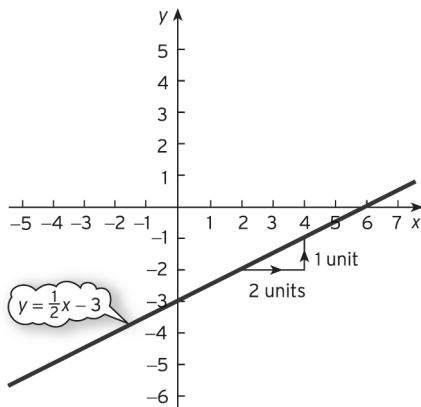


Figure S1.11

Section 1.4 Algebraic solution of simultaneous linear equations

Practice Problems

1. (a) *Step 1*

It is probably easiest to eliminate y . This can be done by subtracting the second equation from the first:

$$\begin{array}{r} 3x - 2y = 4 \\ x - 2y = 2 - \\ \hline 2x \quad = 2 \end{array}$$

Step 2

The equation $2x = 2$ has solution $x = 2/2 = 1$.

Step 3

If this is substituted into the first equation then

$$\begin{aligned} 3(1) - 2y &= 4 \\ 3 - 2y &= 4 \\ -2y &= 1 \text{ (subtract 3 from both sides)} \\ y &= -\frac{1}{2} \text{ (divide both sides by -2)} \end{aligned}$$

Step 4

As a check the second equation gives

$$\begin{aligned} x - 2y &= 1 - 2(-1/2) \\ &= 1 - (-1) = 2 \checkmark \end{aligned}$$

Hence the solution is $x = 1, y = -1/2$.

If you decide to eliminate x then the corresponding steps are as follows:

Step 1

Triple the second equation and subtract from the first:

$$\begin{array}{r} 3x - 2y = 4 \\ 3x - 6y = 6 - \\ \hline 4y = -2 \end{array}$$

Step 2

The equation $4y = -2$ has solution $y = -2/4 = -1/2$.

Step 3

If this is substituted into the first equation then

$$\begin{aligned} 3x - 2(-\frac{1}{2}) &= 4 \\ 3x + 1 &= 4 \\ 3x &= 3 \end{aligned}$$

(subtract 1 from both sides)

$$x = 1$$

(divide both sides by 3)

(b) Step 1

It is immaterial which variable is eliminated. To eliminate x multiply the first equation by 5, multiply the second by 3 and add:

$$\begin{array}{r} 15x + 25y = 95 \\ -15x + 6y = -33 + \\ \hline 31y = 62 \end{array}$$

Step 2

The equation $31y = 62$ has solution $y = 62/31 = 2$.

Step 3

If this is substituted into the first equation then

$$3x + 5(2) = 19$$

$$3x + 10 = 19$$

$$3x = 9$$

(subtract 10 from both sides)

$$x = 3$$

(divide both sides by 3)

Step 4

As a check the second equation gives

$$-5x + 2y = -5(3) + 2(2)$$

$$= -15 + 4 = -11 \quad \checkmark$$

Hence the solution is $x = 3, y = 2$.

2. (a) **Step 1**

To eliminate x multiply the first equation by 4, multiply the second equation by 3 and add:

$$12x + 24y = -8$$

$$\underline{-12x + 24y = -3 +}$$

$$0y = -11$$

Step 2

This is impossible, so there are no solutions.

(b) **Step 1**

To eliminate x multiply the first equation by 2 and add to the second:

$$-10x + 2y = 8$$

$$\underline{10x - 2y = -8 +}$$

$$0y = 0$$

Step 2

This is true for any value of y , so there are infinitely many solutions.

3. **Step 1**

To eliminate x from the second equation multiply equation (2) by 2 and subtract from equation (1):

$$\begin{array}{r}
 2x + 2y - 5z = -5 \\
 2x - 2y + 2z = 6 - \\
 \hline
 4y - 7z = -11
 \end{array} \tag{4}$$

To eliminate x from the third equation multiply equation (1) by 3, multiply equation (3) by 2 and add:

$$\begin{array}{r}
 6x + 6y - 15z = -15 \\
 -6x + 2y + 4z = -4 + \\
 \hline
 8y - 11z = -19
 \end{array} \tag{5}$$

The new system is

$$\begin{array}{l}
 2x + 2y - 5z = -5 \\
 4y - 7z = -11 \\
 8y - 11z = -19
 \end{array}$$

Step 2

To eliminate y from the third equation multiply equation (4) by 2 and subtract equation (5):

$$\begin{array}{r}
 8y - 14z = -22 \\
 8y - 11z = -19 \\
 \hline
 -3z = -3
 \end{array} \tag{6}$$

The new system is

$$\begin{array}{l}
 2x + 2y - 5z = -5 \tag{1} \\
 4y - 7z = -11 \tag{4} \\
 -3z = -3 \tag{6}
 \end{array}$$

Step 3

Equation (6) gives $z = -3/-3 = 1$. If this is substituted into equation (4) then

$$\begin{array}{l}
 4y - 7(1) = -11 \\
 4y - 7 = -11 \\
 4y = -4 \text{ (add 7 to both sides)} \\
 y = -1 \text{ (divide both sides by 4)}
 \end{array}$$

Finally, substituting $y = -1$ and $z = 1$ into equation (1) produces

$$\begin{array}{l}
 2x + 2(-1) - 5(1) = -5 \\
 2x - 7 = -5 \\
 2x = 2 \text{ (add 7 to both sides)} \\
 x = 1 \text{ (divide both sides by 2)}
 \end{array}$$

Step 4

As a check the original equations (1), (2) and (3) give

$$2(1) + 2(-1) - 5(1) = -5 \quad \checkmark$$

$$1 - (-1) + 1 = 3 \quad \checkmark$$

$$-3(1) + (-1) + 2(1) = -2 \quad \checkmark$$

Hence the solution is $x = 1, y = -1, z = 1$.

Exercise 1.4 (p. 61)

(a) Step 1

Add the two equations to eliminate x :

$$\begin{array}{r} -2x + y = 2 \\ 2x + y = -6 + \\ \hline 2y = -4 \end{array}$$

Step 2

The equation $2y = -4$ has solution $y = -4/2 = -2$.

Step 3

If this is substituted into the first equation then

$$-2x + (-2) = 2$$

$$-2x - 2 = 2$$

$$-2x = 4$$

$$\text{so } \frac{4}{-2} = -2$$

Step 4

As a check the second equation gives

$$2x + y = 2(-2) + (-2)$$

$$= -4 - 2 = -6 \quad \checkmark$$

Hence the solution is $x = -2, y = -2$

(b) Step 1

It is probably easiest to eliminate y . This can be done by subtracting the second equation from the first:

$$\begin{array}{r} 3x + 4y = 12 \\ x + 4y = 8 - \\ \hline 2x = 4 \end{array}$$

Step 2

The equation $2x = 4$ has solution $x = 4/2 = 2$.

Step 3

If this is substituted into the first equation then

$$\begin{array}{l} 3(2) + 4y = 12 \\ 6 + 4y = 12 \\ 4y = 6 \text{ (subtract 6 from both sides)} \\ y = \frac{6}{4} = 3/2 \text{ (divide both sides by 4)} \end{array}$$

Step 4

As a check the second equation gives

$$\begin{array}{l} x + 4y = 2 + 4(3/2) \\ = 2 + 6 = 8 \quad \checkmark \end{array}$$

Hence the solution is $x = 2, y = 3/2$

(c) Step 1

To eliminate x multiply the first equation by 2, subtract the second :

$$\begin{array}{r} 4x + 2y = 8 \\ 4x - 3y = 3 - \\ \hline 5y = 5 \end{array}$$

Step 2

The equation $5y = 5$ has solution $y = 5/5 = 1$.

Step 3

If this is substituted into the first equation then

$$2x + 1 = 4$$

$$2x = 3$$

$$\text{so } x = \frac{3}{2}$$

Step 4

As a check the second equation gives

$$4x - 3y = 4 \quad \frac{3}{2} - 3(1)$$

$$= 6 - 3 = 3 \quad \checkmark$$

Hence the solution is $x = 3/2, y = 1$

(d) Step 1

To eliminate x multiply the first equation by 6, and subtract the second equation:

$$6x + 6y = 6$$

$$6x + 5y = 15 -$$

$$\hline y = -9$$

Step 2

$$y = -9$$

Step 3

If this is substituted into the first equation then

$$x - 9 = 1$$

$$x = 10$$

Step 4

As a check the second equation gives

$$6x + 5y = 6(10) + 5(-9)$$

$$= 60 - 45 = 15 \quad \checkmark$$

Hence the solution is $x = 10, y = -9$.

2. The lines are sketched in Figure S1.12.

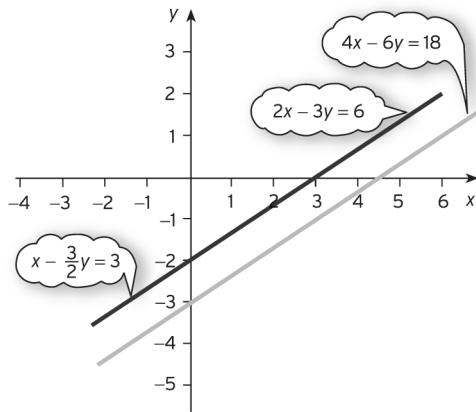


Figure S1.12

(a) The two lines are on top of each other so intersect throughout their length. There are infinitely many solutions.

(b) The lines are parallel so do not intersect anywhere. There is no solution.

3. (a) *Step 1*

To eliminate x multiply the first equation by 3, and add:

$$\begin{array}{r} -9x + 15y = 12 \\ 9x - 15y = -12 - \\ \hline 0y = 0 \end{array}$$

Step 2

The equation $0y = 0$ is true for all values of y so there are infinitely many solutions

(b) *Step 1*

To eliminate x multiply the first equation by 5, multiply the second by 2 and subtract :

$$\begin{array}{r} 30x - 10y = 15 \\ 30x - 10y = 8 - \\ \hline 0y = 7 \end{array}$$

Step 2

The equation $0y = 7$ is not true any value of y so there are no solutions.

4. Step 1

To eliminate x multiply the second equation by 2, and add to the first:

$$\begin{array}{r} 6x - 4y = 2 \\ -6x + 4y = 2k + 2 \\ \hline 0y = 2k + 2 \end{array}$$

Step 2

The equation $0y = 2k + 2$ only has solutions (when there will be infinitely many) if the right-hand side is zero so that

$$2k + 2 = 0$$

$$2k = -2 \quad (\text{subtract 2 from both sides})$$

$$k = -1$$

Section 1.5 Supply and demand analysis

Practice Problems

1. (a) 0 (b) 48 (c) 16 (d) 25 (e) 1 (f) 17

The function g reverses the effect of f and takes you back to where you started. For example, if 25 is put into the function f , the outgoing number is 0; and when 0 is put into g , the original number, 25, is produced. We describe this by saying that g is the inverse of f (and vice versa).

2. The demand curve that passes through $(0, 75)$ and $(25, 0)$ is sketched in Figure S1.13. From this diagram we see that

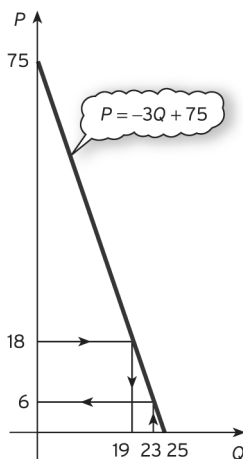


Figure S1.13

(a) $P = 6$ when $Q = 23$

(b) $Q = 19$ when $P = 18$

Alternatively, using algebra:

(a) Substituting $Q = 23$ gives

$$P = -3(23) + 75 = 6$$

(b) Substituting $P = 18$ gives $18 = -3Q + 75$ with solution $Q = 19$

3. (a) In equilibrium, $Q_S = Q_D = Q$, so

$$P = -4Q + 120$$

$$P = \frac{1}{3}Q + 29$$

Hence

$$-4Q + 120 = \frac{1}{3}Q + 29 \quad (\text{since both sides equal } P)$$

$$-4\frac{1}{3}Q + 120 = 29 \quad (\text{subtract } \frac{1}{3}Q \text{ from both sides})$$

$$-4\frac{1}{3}Q = -91 \quad (\text{subtract } 120 \text{ from both sides})$$

$$Q = 21 \quad (\text{divide both sides by } -4\frac{1}{3})$$

Substituting this value into either the demand or supply equations gives $P = 36$.

(b) After the imposition of a \$13 tax the supply equation becomes

$$P - 13 = \frac{1}{3}Q_S + 29$$

$$P = \frac{1}{3}Q_S + 42$$

The demand equation remains unchanged, so, in equilibrium,

$$P = -4Q + 120$$

$$P = \frac{1}{3}Q + 42$$

Hence

$$-4Q + 120 = \frac{1}{3}Q + 42$$

This equation can now be solved as before to get $Q = 18$ and the corresponding price is $P = 48$. The equilibrium price rises from \$36 to \$48, so the consumer pays an additional \$12. The remaining \$1 of the tax is paid by the firm.

4. For good 1, $Q_{D_1} = Q_{S_1} = Q$ in equilibrium, so the demand and supply equations become

$$\begin{aligned} Q_1 &= 40 - 5P_1 + P_2 \\ Q_1 &= -3 + 4P_1 \end{aligned}$$

Hence

$$\begin{aligned} 40 - 5P_1 - P_2 &= -3 + 4P_1 && \text{(since both sides equal } Q_1) \\ 40 - 9P_1 - P_2 &= -3 && \text{(subtract } 4P_1 \text{ from both sides)} \\ -9P_1 - P_2 &= -43 && \text{(subtract 40 from both sides)} \end{aligned}$$

For good 2, $Q_{D_2} = Q_{S_2}$ in equilibrium, so the demand and supply equations become

$$\begin{aligned} Q_2 &= 50 - 2P_1 + 4P_2 \\ Q_2 &= -7 + 3P_2 \end{aligned}$$

Hence

$$\begin{aligned} 50 - 2P_1 - 4P_2 &= -7 + 3P_2 && \text{(since both sides equal } Q_2) \\ 50 - 2P_1 - 7P_2 &= -7 && \text{(subtract } 3P_2 \text{ from both sides)} \\ -2P_1 - 7P_2 &= -57 && \text{(subtract 50 from both sides)} \end{aligned}$$

The equilibrium prices therefore satisfy the simultaneous equations

$$-9P_1 - P_2 = -43 \tag{1}$$

$$-2P_1 - 7P_2 = -57 \tag{2}$$

Step 1

Multiply equation (1) by 2 and (2) by 9 and subtract to get

$$61P_2 = 427 \tag{3}$$

Step 2

Divide both sides of equation (3) by 61 to get $P_2 = 7$.

Step 3

Substitute P_2 into equation (1) to get $P_1 = 4$.

If these equilibrium prices are substituted into either the demand or the supply equations then $Q_1 = 13$ and $Q_2 = 14$.

The goods are complementary because the coefficient of P_2 in the demand equation for good 1 is negative, and likewise for the coefficient of P_1 in the demand equation for good 2.

Exercise 1.5 (p. 75)

1. (a) 21 (b) 45 (c) 15 (d) 2 (e) 10 (f) 0

The answers to parts (a) and (b) show that putting 2 into f gives 21 and putting 21 into g takes us back to 2.

A similar property holds for parts (b) and (e) as well as (c) and (f).

If one function undoes the result of another and takes you back to number you first thought of then we say that they are inverses of each other.

2. The supply curve is sketched in Figure S1.14.

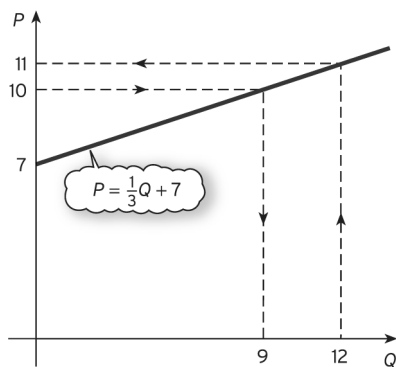


Figure S1.14

- (a) 11; this value can either be found by substituting $Q = 12$ into $P = \frac{1}{3}Q + 7$ or by reading values off the graph.
- (b) 9; this value can either be found by solving the equation, $\frac{1}{3}Q + 7 = 10$ or by reading values off the graph.
- (c) 0; once the price falls below 7 the graph shows that the firm does not plan to produce any goods.
3. (a) $Q = 100 - 10 + 2 \times 40 + \frac{1}{2} \times 6 = 173$.

To work out the new advertising expenditure you could solve the equation

$$100 - 10 + 2 \times 40 + \frac{1}{2}A = 179$$

which gives:

$$170 + \frac{1}{2}A = 179$$

$$\frac{1}{2}A = 9 \quad (\text{subtract } 170 \text{ from both sides})$$

$$A = 18 \quad (\text{multiply both sides by } 2)$$

Hence additional advertising expenditure is $18 - 6 = 12$.

Alternatively notice that the demand increases by $179 - 173 = 6$. In the demand function, the term involving advertising expenditure is $\frac{1}{2}A$ so that for

every one unit increase in A , demand increases by $\frac{1}{2}$. Hence expenditure will need to rise by 12 to achieve a change of 6 in the value of Q .

(b) Superior; the coefficient of Y is positive so an increase in Y leads to an increase in Q .

4. (a) $Q = 30 - 3 \times 4 + 5 = 23$

(b) Substitutable; the coefficient of P_A is positive so a rise in P_A leads to an increase in Q

(c) The demand equation is:

$$30 - 3P + 11 = 23$$

$$41 - 3P = 23$$

$$-3P = -18 \quad (\text{subtract } 41 \text{ from both sides})$$

$$P = 6 \quad (\text{divide both sides by } -3)$$

5. (a) Substituting $Q = 0$ and $Q = 50$ into the supply function give $P = 20$ and 45 respectively.

To find the value of Q when $P = 25$ solve the equation:

$$\frac{1}{2}Q + 20 = 25$$

$$\frac{1}{2}Q = 5 \quad (\text{subtract } 20 \text{ from both sides})$$

$$Q = 10 \quad (\text{multiply both sides by } 2)$$

Line passes through (0,20), (10, 25) and (50, 45)

(b) Line passing through (50,0) and (0,50)

The lines intersect at the point (20, 30) so the equilibrium values are $Q = 20$ and $P = 30$

(c) As income rises demand increases so the demand curve moves to the right. As it does so the point of intersection moves both to the right and upwards on the page.

Hence price and quantity both increase.

6. The new supply equation is

$$P - 4 = \frac{1}{2}Q_S + 23$$

which rearranges as $P = \frac{1}{2}Q_S + 27$.

In equilibrium, $Q_S = Q_D = Q$ so we need to solve the equations:

$$P = -3Q + 48$$

$$P = \frac{1}{2}Q + 27$$

Hence

$$-3Q + 48 = \frac{1}{2}Q + 27$$

$$-3\frac{1}{2}Q + 48 = 27 \quad (\text{subtract } \frac{1}{2}Q \text{ from both sides})$$

$$-3\frac{1}{2}Q = -21 \quad (\text{subtract 48 from both sides})$$

$$Q = 6 \quad (\text{divide both sides by } -3.5)$$

7. In equilibrium, $Q_{D_1} = Q_{S_1} = Q_1$ and $Q_{D_2} = Q_{S_2} = Q_2$

For good 1 we have

$$100 - 2P_1 + P_2 = -10 + P_1$$

$$100 - 3P_1 + P_2 = -10 \quad (\text{subtract } P_1 \text{ from both sides})$$

$$-3P_1 + P_2 = -110 \quad (\text{subtract 100 from both sides}) \quad (1)$$

For good 2 we have

$$5 + 2P_1 - 3P_2 = -5 + 6P_2$$

$$5 + 2P_1 - 9P_2 = -5 \quad (\text{subtract } 6P_2 \text{ from both sides})$$

$$2P_1 - 9P_2 = -10 \quad (\text{subtract 5 from both sides}) \quad (2)$$

It is probably simplest to eliminate P_2 . This can be done by multiplying equation (1) by 9 and adding equation (2).

This gives $-25P_1 = -1000$ and so $P_1 = 40$.

Substituting this into equation (1):

$$-120 + P_2 = -110$$

$$P_2 = 10 \text{ (add 120 to both sides)}$$

The easiest way of finding the equilibrium quantities is to use the supply equations which give:

$$Q_1 = -10 + 40 = 30$$

$$Q_2 = -5 + 60 = 55$$

8. (a) $Q = -20 \times 8 + 0.04 \times 1000 + 4 \times 15 + 3 \times 30 = 30$

(b) Substitutable; e.g. since coefficient of P_r is positive

(c) $-20P + 0.04 \times 8000 + 4 \times 30 + 3 \times 25 = 235$

$$-20P + 515 = 235$$

$$-20P = -280 \text{ (subtract 515 from both sides)}$$

$$P = 14 \text{ (divide both sides by } -20)$$

(d) (i) $Q = -20P + 0.04 \times 2000 + 4 \times 10 + 3 \times 5$

$$Q = -20P + 135$$

slope = -20 , intercept = 135

(ii) $Q + 20P = 135$ (add $20P$ to both sides)

$$20P = -Q + 135 \text{ (subtract } Q \text{ from both sides)}$$

$$P = -0.05Q + 6.75 \text{ (divide both sides by 20)}$$

slope = -0.05 , intercept = 6.75

Section 1.6 Transposition of formulae

Practice Problems

1. (a) $\frac{1}{2}Q = 4$ (subtract 13 from both sides)

$$Q = 8 \quad \text{(multiply both sides by 2)}$$

(b) $\frac{1}{2}Q = P - 13$ (subtract 13 from both sides)

$$Q = 2(P - 13) \quad \text{(multiply both sides by 2)}$$

$$Q = 2P - 26 \quad \text{(multiply out brackets)}$$

(c) $Q = 2 \times 17 - 26 = 8$

2. (a) square multiply by 6

square root divide by 6

$$6x^2 = y$$

$$x^2 = \frac{y}{6} \quad (\text{divide both sides by 6})$$

$$x = \sqrt{\frac{y}{6}} \quad (\text{square root both sides})$$

(b) multiply by 7 subtract 1 reciprocate

divide by 7 add 1 reciprocate

$$\frac{1}{7x-1} = y$$

$$7x-1 = \frac{1}{y} \quad (\text{reciprocate both sides})$$

$$7x = \frac{1}{y} + 1 \quad (\text{add 1 to both sides})$$

$$x = \frac{1}{7} \frac{1}{y} + 1 \quad (\text{divide both sides by 7})$$

3. (a) $x - ay = cx + y$

$$x = cx + y + ay \quad (\text{add } ay \text{ to both sides})$$

$$x - cx = y + ay \quad (\text{subtract } cx \text{ from both sides})$$

$$(1 - c)x = (1 + a)y \quad (\text{factorise both sides})$$

$$x = \frac{1 + a}{1 - c} y \quad (\text{divide both sides by } 1 - c)$$

(b) $y = \frac{x-2}{x+4}$

$$(x+4)y = x-2 \quad (\text{multiply both sides by } x+4)$$

$$xy + 4y = x - 2 \quad (\text{multiply out the brackets})$$

$$xy = x - 2 - 4y \quad (\text{subtract } 4y \text{ from both sides})$$

$$xy - x = -2 - 4y \quad (\text{subtract } x \text{ from both sides})$$

$$(y-1)x = -2 - 4y \quad (\text{factorise left-hand side})$$

$$x = \frac{-2-4y}{y-1} \quad (\text{divide both sides by } y-1)$$

Exercise 1.6 (p. 86)

1. $2Q + 8 = P$

$2Q = P - 8$ (subtract 8 from both sides)

$Q = \frac{1}{2}(P - 8) = \frac{1}{2}P - 4$ (divide both sides by 2)

Substituting $P = 52$ into this formula gives $Q = \frac{1}{2} \times 52 - 4 = 26 - 4 = 22$

2. (a) $y = 2x + 5$; (b) $y = 2(x + 5)$; (c) $y = \frac{5}{x^2}$;
 (d) $y = 2(x + 4)^2 - 3$.

3. (a) multiply by 5 add 3

(b) add 3 multiply by 5

(c) multiply by 6 subtract 9

(d) square multiply by 4 subtract 6

(e) divide by 2 add 7

(f) reciprocate multiply by 2

(g) add 3 reciprocate

4. (a) $9x - 6 = y$

$9x = y + 6$ (add 6 to both sides)

$x = \frac{1}{9}(y + 6)$ (divide both sides by 9)

(b) $\frac{x + 4}{3} = y$

$x + 4 = 3y$ (multiply both sides by 3)

$x = 3y - 4$ (subtract 4 from both sides)

(c) $\frac{x}{2} = y$

$x = 2y$ (multiply both sides by 2)

(d) $\frac{x}{5} + 8 = y$

$$\frac{x}{5} = y - 8 \text{ (subtract 8 from both sides)}$$

$$x = 5(y - 8) \text{ (multiply both sides by 5)}$$

(e) $y = \frac{1}{x+2}$

$$y(x+2) = 1 \text{ (multiply both sides by } x+2 \text{)}$$

$$x+2 = \frac{1}{y} \text{ (divide both sides by } y \text{)}$$

$$x = \frac{1}{y} - 2; \text{ (subtract 2 from both sides)}$$

(f) $y = \frac{4}{3x-7}$

$$y(3x-7) = 4 \text{ (multiply both sides by } 3x-7 \text{)}$$

$$3x-7 = \frac{4}{y} \text{ (divide both sides by } y \text{)}$$

$$3x = \frac{4}{y} + 7 \text{ (add 7 to both sides)}$$

$$x = \frac{1}{3} \left(\frac{4}{y} + 7 \right) \text{ (divide both sides by 3)}$$

5. (a) $aP + b = Q$

$$aP = Q - b \text{ (subtract } b \text{ from both sides)}$$

$$P = \frac{1}{a}(Q - b) = \frac{Q}{a} - \frac{b}{a} \text{ (divide both sides by } a \text{)}$$

(b) $Y = aY + b + I$

$$Y - aY = b + I \text{ (subtract } aY \text{ from both sides)}$$

$$(1-a)Y = b + I \text{ (factorise by taking out a common factor of } Y \text{)}$$

$$Y = \frac{b+I}{1-a} \text{ (divide both sides by } 1-a \text{)}$$

$$(c) \quad Q = \frac{1}{aP+b}$$

$$Q(aP+b) = 1 \quad (\text{multiply both sides by } aP+b)$$

$$aP+b = \frac{1}{Q} \quad (\text{divide both sides by } Q)$$

$$aP = \frac{1}{Q} - b \quad (\text{subtract } b \text{ from both sides})$$

$$P = \frac{1}{a} \left(\frac{1}{Q} - b \right) = \frac{1}{aQ} - \frac{b}{a} \quad (\text{divide both sides by } a)$$

$$6. \quad y = \frac{3}{x} - 2$$

$$y+2 = \frac{3}{x} \quad (\text{add } 2 \text{ to both sides})$$

$$(y+2)x = 3 \quad (\text{multiply both sides by } x)$$

$$x = \frac{3}{y+2} \quad (\text{divide both sides by } y+2)$$

Section 1.7 National income determination

Practice Problems

$$1. \quad S = Y - C$$

$$= Y - (0.8Y + 25) \quad (\text{substitute expression for } C)$$

$$= Y - 0.8Y - 25 \quad (\text{multiply out brackets})$$

$$= 0.2Y - 25 \quad (\text{collect terms})$$

$$2. \quad Y = C + I \quad (\text{from theory})$$

$$C = 0.8Y + 25 \quad (\text{given in question})$$

$$I = 17 \quad (\text{given in question})$$

Substituting the given value of I into the first equation gives

$$Y = C + 17$$

and if the expression for C is substituted into this then

$$Y = 0.8Y + 42$$

$$0.2Y = 42 \quad (\text{subtract } 0.8Y \text{ from both sides})$$

$$Y = 210 \quad (\text{divide both sides by } 0.2)$$

Repeating the calculations with $I = 18$ gives $Y = 215$, so a 1 unit increase in investment leads to a 5 unit increase in income. The scale factor, 5, is called the investment multiplier. In general, the investment multiplier is given by $1/(1 - a)$, where a is the marginal propensity to consume. The foregoing is a special case of this with $a = 0.8$.

3. $Y = C + I + G \quad (1)$

$$G = 40 \quad (2)$$

$$I = 55 \quad (3)$$

$$C = 0.8Y_d + 25 \quad (4)$$

$$T = 0.1Y + 10 \quad (5)$$

$$Y_d = Y - T \quad (6)$$

Substituting equations (2) and (3) into equation (1) gives

$$Y = C + 95 \quad (7)$$

Substituting equation (5) into (6) gives

$$Y_d = Y - (0.1Y + 10)$$

$$= 0.9Y - 10$$

so from equation (4),

$$C = 0.8(0.9Y - 10) + 25$$

$$= 0.72Y + 17 \quad (8)$$

Finally, substituting equation (8) into (7) gives

$$Y = 0.72Y + 112$$

which has solution $Y = 400$.

4. The commodity market is in equilibrium when

$$Y = C + I$$

so we can substitute the given expressions for consumption ($C = 0.7Y + 85$) and investment

($I = 50r + 1200$) to deduce that

$$Y = 0.7Y - 50r + 1285$$

which rearranges to give the IS schedule,

$$0.3Y + 50r = 1285 \quad (1)$$

The money market is in equilibrium when

$$M_S = M_D$$

Now we are given that $M_S = 500$ and that total demand,

$$M_D = L_1 + L_2 = 0.2Y - 40r + 230$$

so that

$$500 = 0.2Y - 40r + 230$$

which rearranges to give the LM schedule,

$$0.2Y - 40r = 270 \quad (2)$$

We now solve equations (1) and (2) as a pair of simultaneous equations.

Step 1

Multiply equation (1) by 0.2 and (2) by 0.3 and subtract to get

$$22r = 176$$

Step 2

Divide through by 22 to get $r = 8$.

Step 3

Substitute $r = 8$ into equation (1) to give $Y = 2950$.

The IS and LM curves shown in Figure S1.15 confirm this, since the point of intersection has coordinates (8, 2950). A change in I does not affect the LM schedule. However, if the autonomous level of investment increases from its current level of 1200 then the right-hand side of the IS schedule (1) will rise. The IS curve moves upwards, causing both r and Y to increase.

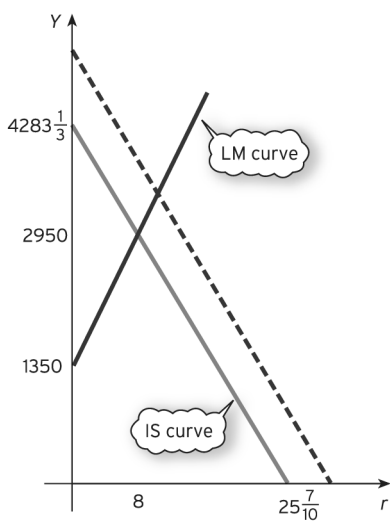


Figure S1.15

Exercise 1.7 (p. 101)

1. (a) 40; (b) 0.7

$$0.7Y + 40 = C$$

$$0.7Y = C - 40 \quad (\text{subtract 40 from both sides})$$

$$\frac{7}{10}Y = C - 40$$

$$7Y = 10(C - 40) \quad (\text{multiply both sides by 10})$$

$$Y = \frac{10}{7}(C - 40) \quad (\text{divide both sides by 7})$$

$$Y = \frac{10}{7}(110 - 40) = 100$$

2. (a) $S = Y - C = Y - (0.9Y + 72) = Y - 0.9Y - 72 = 0.1Y - 72$

(b) $S = Y - C = Y - (0.8Y + 100) = Y - 0.8Y - 100 = 0.2Y - 100$

3. (a) $Y = C + I$

$$Y = 0.6Y + 30 + 100$$

$$Y = 0.6Y + 130$$

$$0.4Y = 130 \quad (\text{subtract } 0.6Y \text{ from both sides})$$

$$Y = 325 \quad (\text{divide both sides } 0.4)$$

(b) $C = 0.6Y + 30 = 0.6 \times 325 + 30 = 225$

(c) $S = Y - C = 325 - 225 = 100$

4. $10a + b = 28$

$$30a + b = 44$$

Subtracting these equations gives:

$$20a = 16 \text{ so } a = 0.8$$

From the first equation,

$$10 \times 0.8 + b = 28$$

$$8 + b = 28$$

$$b = 20$$

Finding the equilibrium level of national income:

$$Y = C + I = 0.8Y + 20 + 13 = 0.8Y + 33$$

$$0.2Y = 33$$

$$Y = 165$$

5. $Y = C + I + G$

$$Y = 0.75Y_d + 45 + 40 + 50$$

$$Y = 0.75Y_d + 135$$

$$\text{Now } Y_d = Y - T = Y - (0.2Y + 80) = Y - 0.2Y - 80 = 0.8Y - 80$$

Substituting gives

$$Y = 0.75(0.8Y - 80) + 135 = 0.6Y - 60 + 135 = 0.6Y + 75$$

Hence

$$0.4Y = 75 \text{ (subtract } 0.6Y \text{ from both sides)}$$

$$Y = 187.5 \text{ (divide both sides by } 0.4)$$