Discounted Cash Flows

Present Value Calculations (Exhibit 1)

The fundamental idea of finance is that a dollar in the future is worth less than a dollar today. This rule is implemented by using a discount rate to convert future payments to their current (present) value. Conversely, the interest rate is used to compute the future value of a current amount (the term "discount rate" is typically used when computing the present value of future cash flows; the term "interest rate" is typically used when computing the future value of a current amount).

Value of an investment

Suppose we invest $100,000 at a 10% annual interest rate. As shown in Example 1 of Exhibit 1, after one year the investment grows to $110,000 ($100,000 × 1.1). After two years it grows to $121,000 ($110,000 at the end of year one, × 1.1). At the end of the third year the investment grows to $133,100 ($121,000 at the end of year two, × 1.1). By the end of the eighth year the investment is worth $214,359.

An easier way to determine the value after some period is to simply multiply the initial investment by 1 plus the interest rate, raised to the number of years of the investment; Example 2 calculates the year-end values after years one through eight using that method. For example, at the end of year eight the investment would be worth $214,359, or $100,000 × (1.1) ^ 8.

Present value of a future cash flow

Example 3 shows estimated cash flows from investing in a new project. The discount rate is 12%, so the present value of the first cash flow is $22,321' [$25,000/(1 + discount rate)]. The present value of the second cash flow is $31,888; [$40,000/(1 + discount rate)^2], and so on. The present value of the cash flows is simply the sum of the present value of the individual cash flows, or $824,522. The present values of a set of cash flows can also be computed using Excel's NPV function, as described in the next section.

Excel Present Value Calculations (Exhibit 2)

Two Excel functions are widely used in accounting and finance, PMT and NPV.

PMT

The payment function computes the periodic payment needed to repay a present value over some period at a given interest rate. The Excel entry, as shown in Table A of Exhibit 2, is =pmt(rate, number of periods, present value, [optional future value], and [optional type]). Example 1 in Table A computes the monthly payment on a 15-year (180 month), $300,000 fixed rate mortgage at a monthly interest rate of 0.5%. The default type for the payment function is type 0, which assumes that payments are made at the end of a month A.

If you wish to compute the required payment with payments at the beginning of a period, you must enter "1" as the fifth parameter [type]. For Excel to know the 1 is the fifth and not the fourth parameter, you need to enter a 0 for the fourth parameter, which is the future value [fv]. As shown in Example 2 of Table A, paying at the beginning of the month instead of the end of the month reduces monthly payments from $2,532 to $2,519. Example 3 shows that if you wish to repay the loan and have $50,000 at the end of 15 years (future value of $50,000), you must pay $2,690 at the beginning of each month. In practice, it is rare to enter a future value in the PMT function.

NPV

The NPV function computes the present value of a set of cash flows at a specified discount rate: =npv(rate, range of values). Payments must be equally spaced, such as monthly, quarterly, or annually; otherwise, the XNPV function must be used. In practice, nearly all payments are considered equally spaced. When you specify a range, every cell in the range must have a value or a zero—the NPV function ignores blank rows. Assume you want to discount the cash flows shown in Example 1 of Table B at a 6% annual discount rate. Assume there is a negative cash flow in year 1, a zero cash flow in year 2, and the remaining cash flows as shown. The calculation in Example 1 shows an NPV of $780,630. Example 2, which moves the cash flows in years 3 through 10 to years 2 through 9, shows the same NPV as Example 1 because Excel ignored the blank entry for year 2 in Example 1. Example 3 of Table B shows that by entering $0 in year 2, the NPV declines to $733,774, which is the correct value.

Periodic Interest Rates (Exhibit 2)

The PMT function requires an interest rate, and the NPV function requires a discount rate, which are basically two terms for the same thing. Those rates must be for the period used in the function. If the period for the payments or cash flows are annual, the interest or discount rate must be an annual rate. If payments or cash flows are (1) monthly, such as mortgage payments; (2) quarterly, such as projected quarterly cash flows; or (3) semi-annual, such as bond interest payments, you must convert the annual rate to the relevant period rate.

Most Excel financial functions say to divide the annual interest or discount rate by the number of periods to obtain the period interest rate or period discount rate. For example: compute the period interest rate by dividing the annual interest rate by the number of periods (12 for monthly, 4 for quarterly, and 2 for semi-annual). For an annual interest rate of 6%, the rates would be .5%, 1.5%, or 3%, as shown in Exhibit 2, Table C, Example 1. However, if we compound the .5% monthly rate for 12 periods [$(1.005)^{12}$], as in the third column of Example 1, Table C, the annualized rate is 6.17%, not 6%. Similarly, compounding the 1.5% quarterly rate for four periods gives an annualized interest rate of 6.14%, and compounding the 3% semi-annual rate for two periods gives an annualized interest rate of 6.09%.

The correct procedure is to add 1.00 to the annual interest rate, raise that number to 1 divided by the number of periods, and then subtract 1, as shown in Example 2 of Table C. With that method, compounding the computed rate by the number of periods always produces the correct annual interest rate of 6% (however, by convention, bonds interest rates are quoted as yield-to-maturity, where the semi-annual interest rate is one-half the annual interest rate).

Automobile Financing

Suppose you decide to purchase a new automobile for $24,150. The dealer's financing agent offers you the following financing alternatives:

Effective Annual	
Interest rate	Loan term
0.0%	24 months
1.5%	30 months
3.0%	36 months

Required

1. Compute the monthly interest rate for the three options.
2. Using Excel's PMT function, compute the monthly payments for each of the three alternatives.
3. Assume your annual cost of funds is 6.0%. Using the monthly payments you computed in question 2, use Excel's NPV function to compute the present value of your cost of buying the automobile under each of the three alternatives. Which of the three alternatives would you prefer? Explain.

Alternative	A	B	C
Automobile cost	$24,150	$24,150	$24,150
Annual interest rate	0.0%	1.5%	3.0%
Monthly interest rate			
Number of periods	24	30	36
Payment per month			
Your annual discount rate	6.0%	6.0%	6.0%
Your monthly discount rate			
Present value			
Payments, by month			
1			
2			
3			
4			
5			

Introduction to Fixed Income Securities (Exhibit 3)

A bond is described by its issuer, coupon, and maturity. Treasury bonds with maturities of 10 years or less are typically called notes; those that mature in more than 10 years are typically called bonds. There is no practical difference between a Treasury bond and a Treasury note; both pay interest semi-annually.

An 8% December 31, 2016, GM bond issued on December 31, 2010, would be a 6-year bond issued by General Motors that pays 4% each June 30 and December 31 until December 31, 2016. On the maturity date, December 31, 2016, the bond pays the last interest payment and repays the principal. For a $1,000 bond, each 6-month interest payment would be $40. "Coupon" refers to how interest was paid in the past.

A "bearer" bond paid interest to whoever had possession of the bond. Each bearer bond had coupons attached; twice each year the owner or owner's agent "clipped" the relevant coupon for that period and exchanged it at a bank for the cash interest payment. In 1982, the U.S. Internal Revenue Service banned bearer bonds. All U.S. bonds are now "registered" bonds that pay interest to whoever is registered with the bond issuer as the owner.

As mentioned earlier, if we pay or receive interest other than annually, we must adjust the annual rate to a period rate. Using the above example, suppose we invest $1,000 on December 31, 2006, and receive 4% interest every six months. If we reinvest the first interest payment at that same 4% interest rate, our investment will be worth $1,000 × 1.04 = $1,040 on June 30 and $1,040 × 1.04 = $1,081.60 on December 31. As a result, our effective interest rate on the 8% note is 8.16%.

By tradition, a bond's quoted annual interest rate is almost always twice the six-month coupon rate and is called the "annual yield to maturity," not the effective annual interest rate. If the *Wall Street Journal* reports that a bond's annual yield to maturity is 8.16%, the six-month interest/discount rate is 4.08% and the effective annual yield is 8.3265% $[(1.0408^2) - 1]$. To compute the present value of a bond's future interest payments and the principal repayment, we discount those cash flows at a semi-annual discount rate that equals one-half of the market's yield-to-maturity.

Examples 1, 2, and 3 in Exhibit 3 show cash flows for a $50 million issue of 8% notes. It also shows the present values when the market's yield to maturity is 8.00%, 7.50%, and 8.50%. If we issue an 8% bond when the market's yield to maturity for similar bonds is 8.00%, we receive $50,000,000. If the market's yield to maturity is 7.50%, we receive $51,190,336.74 for the bonds and if the market's yield to maturity is 8.50%, we receive only $49,843,700.64.

Required

1. Using the examples from Exhibit 3, assume a firm issues $200 million of 5-year 3.5% notes on December 31, 2010 (1.75% interest on the $200 million payable every six months for five years, with the principal due at maturity). Assume the yield to maturity on December 31, 2010, was 3.63%. Show the cash flows for the notes issued and also compute the amount the firm would receive from issuing the 3.5% notes at the 3.63% yield to maturity.

2. Assume the yield to maturity when the notes were issued was 3.42%. Compute the amount the firm would receive from selling $200 million of five-year 3.5% notes at a 3.42% yield to maturity.

EXHIBIT 1

PRESENT VALUE
CALCULATIONS

Example 1

Interest rate		10.00%
Year		**Multiplication factor**
0	$100,000	
1	$110,000	1.100
2	$121,000	1.100
3	$133,100	1.100
4	$146,410	1.100
5	$161,051	1.100
6	$177,156	1.100
7	$194,872	1.100
8	$214,359	1.100

Example 2

Interest rate		10.00%
Year		**Compound factor**
0	$100,000	
1	$110,000	1.100
2	$121,000	1.210
3	$133,100	1.331
4	$146,410	1.464
5	$161,051	1.611
6	$177,156	1.772
7	$194,872	1.949
8	$214,359	2.144

Example 3

Discount rate		12.00%	
Year	**Cash flows**	**Discount factor**	**Present value of cash flow**
1	$25,000	1.120	$22,321
2	$40,000	1.254	$31,888
3	$125,000	1.405	$88,973
4	$285,000	1.574	$181,123
5	$325,000	1.762	$184,414
6	$305,000	1.974	$154,522
7	$245,000	2.211	$110,826
8	$125,000	2.476	$50,485
Total present value of cash flows			$824,552

EXHIBIT 2

EXCEL PRESENT
VALUE
CALCULATIONS

Table A

=pmt(rate, nper, pv, [fv], [type]);

	Example 1	Example 2	Example 3
Payment	**$2,532**	**$2,519**	**$2,690**
Rate	0.50%	0.50%	0.50%
nper	180	180	180
pv	−$300,000	−$300,000	−$300,000
fv	omit	$0	−$50,000
type	omit	1	1

Table B

=npv(rate, range or list the values)

	Example 1	Example 2	Example 3
NPV:	**$780,630**	**$780,630**	**$733,774**
Discount rate	6.0%	6.0%	6.0%
Cash flows			
Year			
1	−$50,000	−$50,000	−$50,000
2		$25,000	$0
3	$25,000	$75,000	$25,000
4	$75,000	$95,000	$75,000
5	$95,000	$155,000	$95,000
6	$155,000	$350,000	$155,000
7	$350,000	$250,000	$350,000
8	$250,000	$150,000	$250,000
9	$150,000	$75,000	$150,000
10	$75,000		$75,000

Table C

Example 1

	Periods	Divide by # of periods	(1 + rate)^ # of periods
Annual rate	6.00%		
Monthly	12	0.50%	6.17%
Quarterly	4	1.50%	6.14%
Semi-annual	2	3.00%	6.09%

Example 2

	Periods	(1 + annual)^ (1/periods) − 1	(1 + rate)^ # of periods
Monthly	12	0.49%	6.00%
Quarterly	4	1.47%	6.00%
Semi-annual	2	2.96%	6.00%

EXHIBIT 3

INTRODUCTION
TO FIXED INCOME
SECURITIES (BONDS)

	Example 1	Example 2	Example 3
Coupon rate	8.00%	8.00%	8.00%
Semi-annual coupon	4.00%	4.00%	4.00%
Face value of notes issued	$50,000,000	$50,000,000	$50,000,000
Yield-to-maturity	8.00%	7.50%	8.50%
Semi-annual discount rate	4.00%	3.75%	4.25%
Effective annual interest rate	8.16%	7.64%	8.68%
Market value of notes issued	$50,000,000.00	$51,190,336.74	$48,843,700.64
31-Dec-06			
30-Jun-07	$2,000,000	$2,000,000	$2,000,000
31-Dec-07	$2,000,000	$2,000,000	$2,000,000
30-Jun-08	$2,000,000	$2,000,000	$2,000,000
31-Dec-08	$2,000,000	$2,000,000	$2,000,000
30-Jun-09	$2,000,000	$2,000,000	$2,000,000
31-Dec-09	$2,000,000	$2,000,000	$2,000,000
30-Jun-10	$2,000,000	$2,000,000	$2,000,000
31-Dec-10	$2,000,000	$2,000,000	$2,000,000
30-Jun-11	$2,000,000	$2,000,000	$2,000,000
31-Dec-11	$2,000,000	$2,000,000	$2,000,000
30-Jun-12	$2,000,000	$2,000,000	$2,000,000
31-Dec-12	$52,000,000	$52,000,000	$52,000,000