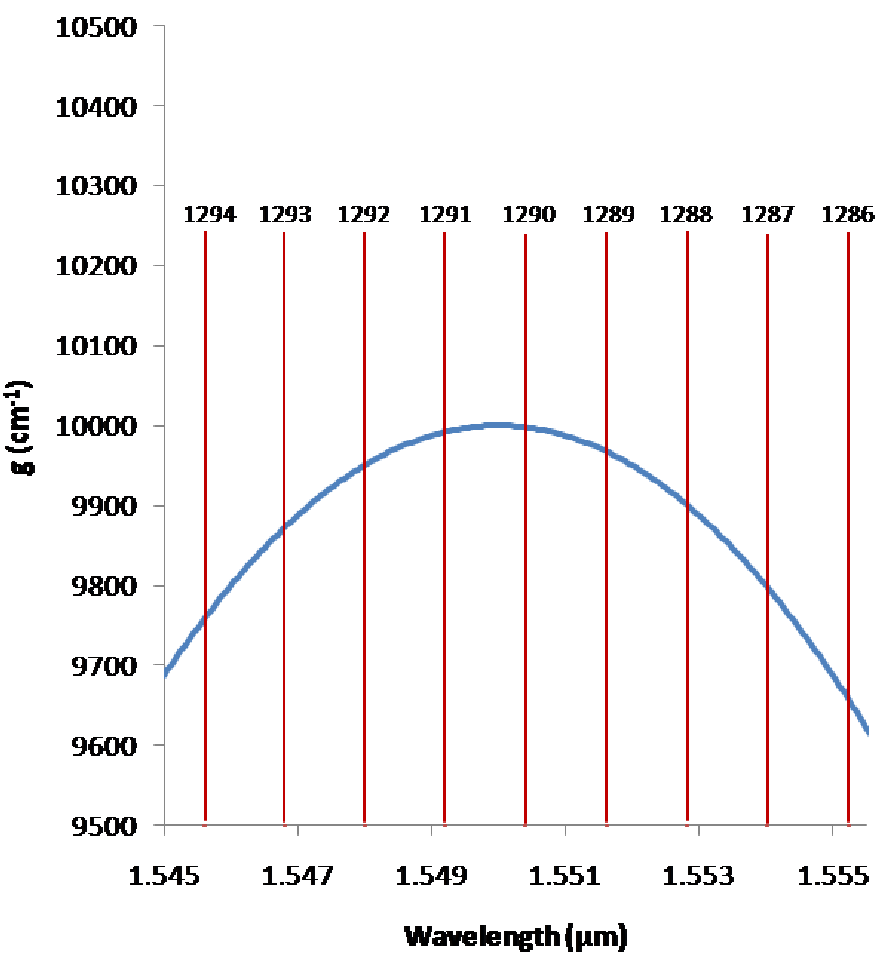
1. The necessary elements of a laser cavity are two mirrors and a gain region. For single wavelength operation, a mode selection filter is also required.

2. To find the lasing wavelength, you need to find the mode with the largest gain.

$$m\lambda = 2nL, \text{ assume } n = 1, L = 1\text{mm}$$

$$g = -1.25\times10^{15}\text{cm}^{-3}(\lambda-1.55\mu\text{m})^2+10^4\text{cm}^{-1}$$

From the plot below, you can see that m = 1290 is closest to the gain peak, and has a wavelength of $\lambda = 1.55039\mu\text{m}$



3.

Advantages:	diode	gas or solid state
higher wall plug efficiency (direct electrical pumping allows higher efficiencies)	$\approx 50\%$	$\approx 1\%$ ($< 10\%$)
longer lifetimes	$\approx 10^6\ h.$	$\approx 10^3\ h.$
higher frequency modulation is possible due to direct electrical pumping	$\approx 10\ GHz$	$\approx 100\ MHz$
small size due to higher gain (≈ 1000 cm^{-1} vs. < 5 cm^{-1})	< 1 mm	> 10 cm
cheaper		
wavelength ranges and size make diode lasers better for fiber optic systems		
Disadvantages:		
larger divergence angle due to the small cross sectional area makes collimating optics necessary for free space applications.	$\approx 30^o$	$\approx 0.1^o$
elliptical mode shape is less desirable than circular TEM mode of gas and solid state lasers		
larger linewidth (less coherence) due to larger gain bandwidth (atomic transition has narrower linewidth than band to band transition)		
wavelength is more reproducable	$> 1\ \mathring{A}$	small
larger temperature sensitivity than the atomic transitions of solid state lasers	$\approx 5\mathring{A}/C$	small
lower power		
more limited wavelength range		
more difficult to tune wavelength		
energy storage is smaller due to smaller cavity volume		

4. Some common applications of a laser diode are CD/DVD players, Blu-Ray Players, laser printers, bar-code scanners and laser pointers. Other applications are optical mice, LIDAR, pump lasers for Er-doped fiber amplifiers, optical coherence tomography, and many others.

5. In solid state lasers, the energy levels of atoms are only slightly perturbed by the surrounding gas or host atoms, and remain effectively unchanged from the isolated atom. So, the emitted or absorbed photon needs to have a very specific energy. In diode lasers, the semiconductor materials are covalently bonded, and atomic levels of the atoms actually broaden into bands of allowed levels because of the perturbations from the nearby atoms, and the valence and conduction bands are formed. This gives a distribution of emitted photon energies.

6. Spontaneous Recombination: An electron in the conduction band recombines spontaneously with a hole in the valence band and generates a photon.

Stimulated Recombination: An incoming photon will perturb an electron in the conduction band and it will recombine with a hole in the valence band, emitting another photon; the incoming photon is not destroyed in this process. This leads to multiplication of photons and lasing.

Stimulated Generation: A photon is absorbed and the energy from the photon stimulates an electron in the valence band to jump to the conduction band, leaving behind a hole in the valence band.

Nonradiative Recombination: Recombination of an electron and hole in a way such that no photon is emitted; rather the energy from recombination dissipates in the form of heat into the crystal lattice. This can occur through the presence of traps or Auger recombination

7. In practice, there are two general nonradiative mechanisms for carriers that are important. The first involves nonradiative recombination centers, such as point defects, surfaces, and interfaces, in the active region of the laser. To be effective, these do not require the simultaneous existence of electrons and holes or other particles. Thus, the recombination rate via this path tends to be directly proportional to the carrier density, N. The second mechanism is Auger recombination, in which the electron‑hole recombination energy, E_{21}, is given to another electron or hole in the form of kinetic energy.
Thus, again for undoped active regions in which the electron and hole densities are equal, Auger recombination tends to be proportional to N^3 because we must simultaneously have the recombining electron‑hole pair and the third particle that receives the ionization energy.

8. A double heterostructure is formed from a smaller bandgap material being sandwiched between two larger bandgap materials. The discontinuities between the smaller bandgap material and larger bandgap material aid in electron and hole confinement within the smaller bandgap material. The larger bandgap material also has a lower index of refraction than the smaller bandgap material, which increases the confinement factor by confining the optical mode to the smaller bandgap region, where the carriers are present. The Nobel Prize winners for this invention were Zhores I. Alferov and Herbert Kroemer.

9. In direct bandgap double heterostructures, carriers are confined inside the PIN junction, and most of them recombine inside the I region, in transit between P and N. In a regular PN diode, very few carriers recombine in the junction, and they are just swept across the junction by the electric field.

10. Assuming barriers of 8nm (since this was not explicitly given in the problem, any value assumed was correct),

a)

$$n_2 = \frac{8*5*2.77 + 8*4*2.52}{8*5 + 8*4} = 2.66$$

$$n_1 = n_3 = 2.52$$

$$V = k_0 d\left(n_{II}^2 - n_{III}^2\right)^{1/2} = \frac{2\pi}{410}(72)\left(2.66^2 - 2.52^2\right)^{1/2} = 0.94$$

$$b \approx 1 - \frac{\ln\left(\frac{V^2}{2} + 1\right)}{\frac{V^2}{2}} \approx .172$$

$$\bar{n} = \left[b n_{II}^2 + (1-b) n_{III}^2\right]^{1/2} = 2.55$$

b)

$$\gamma^2 = \beta^2 - k_0^2 n_I^2 = k_0^2\left(\bar{n}^2 - n_I^2\right) = \left(\frac{2\pi}{410}\right)^2\left(2.55^2 - 2.52^2\right) = .000036$$

$$\gamma = .00598\ /\ nm$$

$$U \propto \exp^{-\gamma x}$$

11.

i) bulk material is normally thicker than quantum wells.

ii) For quantum wells, carriers are confined in the wells, so the carrier injection efficiency is higher.

iii) The density of states are different. Quantum wells have separate energy states, while the energy states of bulk material are quasi-continuous.

12.

i) They need to lattice matched to avoid a lot of defects.

ii) The cladding material must have a larger bandgap and smaller refraction index at working wavelength, compared to the waveguide material.

13.

i) Si has an indirect bandgap, and most of the III-V materials, such as GaN, InP, GaAs, have direct bandgap. For indirect bandgap material, phonons are generally needed to generate photons because of the different crystal momentum between conduction band valance band. So their efficiency to generate light is very low.

14. GaAs/AlGaAs – datacom lasers in the ~700-900nm range, a lot of them vertical cavity. InGaAsP – in-plane lasers for telecom and sensing, 1.3-1.6um.

15. The main material parameter controlling the current leakage in a double heterostructure or a quantum well is the conduction band offset. The band offset will determine the height of a quantum well in a laser active region. Due to their much lighter effective mass, electrons require much tighter confinement with increasing temperature than holes. Thus, it is important to have high band offset in the conduction band. For GaAs/AlGaAs, 2/3 in the conduction band, for InP/InGaAsP, 40% in the conduction band.

16.

a) Solve using the equations in Appendix 1.

$$E_1^{\infty} = \frac{\hbar^2\pi^2}{2md^2}$$

$$E_1^{\infty} = (3.76 \text{ meV}) \left(\frac{100 \text{ Å}}{50 \text{ Å}}\right)^2$$

$$E_1^{\infty} = 15.04 \text{ meV}$$

$$V_0 = 100 \text{ meV}$$

$$n_{max} = \sqrt{\frac{V_0}{E_1^{\infty}}} = 2.57$$

Since $2 \leq n_{max} < 3$, there are 3 bound states.

---

b) Using Figure A1.4, we can determine what the quantum numbers are for these states:

$$n_{QW1} = 0.8$$
$$n_{QW2} = 1.6$$
$$n_{QW3} = 2.3$$

$$E_n = E_1^{\infty}\, n_{QW}^2$$

Thus,

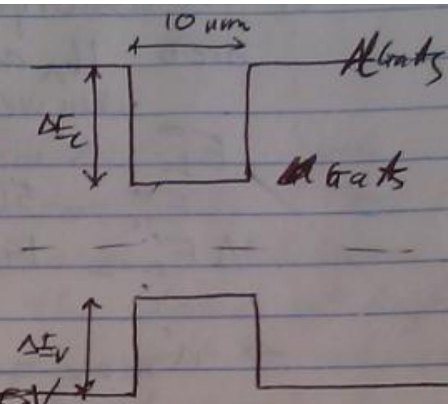
$$E_1 = 9.63 \text{ meV}$$
$$E_2 = 38.5 \text{ meV}$$
$$E_3 = 79.56 \text{ meV}$$

---

c) If V_0 is increased to 200 meV, then $n_{max} = 3.65$. Since $3 \leq n_{max} < 4$, there are 4 bound states.

17.

1.17 a) Assume $Al_{0.2}Ga_{0.8}As$
(found in the table)
→ $E_g = 1.673 eV$ at RT.
GaAs : $E_g = 1.424 eV$ at RT.
Conduction band offset.



$$\Delta E_c = (E_{g\,AlGaAs} - E_{g\,GaAs}) \times 0.6$$

$$= (1.673 - 1.424) 0.6$$

$$= 0.1494\ eV$$

(used the 60:40 rule to calculate the band offset)

$$\Delta E_V = 0.0996\ eV$$

$$E_{1c}^{\infty} = \frac{\hbar^2 \pi^2}{2ml^2} = 3.76 \left(\frac{m_0}{m}\right)\left(\frac{100Å}{l}\right)^2 \quad , l = 100Å$$

$$= 3.76 \times \frac{1}{0.067} \times \left(\frac{100}{100}\right)^2 = 56.12\ meV$$

the maximum quantum number is:

$$n_{maxc} = \sqrt{\frac{\Delta E_c}{E_{1c}^{\infty}}} = \sqrt{\frac{149.4}{56.12}} = 1.63$$

→ there are 2 bound energy states exist.

From the fig. A.4 in the reading note, we can see that for $n_{max} = 1.63$, ~~n_{aw} = 0.66~~ $n_{aw} = 0.725$ and 1.4

$\rightarrow E_1 = n'^2_{aw} E_1^{\infty} = 0.725^2 \times 56.12 = 29.5\ meV$

$E_2 = n'^2_{aw} E_2^{\infty} = 1.4^2 \times 56.12 = 110\ meV$

The next level will be in the continuum $\rightarrow$ near 149.94 meV

c) The assumption "free electron mass" is for the prob. 16, and I don't think it's useful to use it here when we don't know the materials

E_{1c}^{∞} is independent to the depth of the well

$E_{1c}^{\infty} = 56.12\ meV$

$\Delta E_c' = 149.4\ meV \times 2 = 298.8\ meV$

$\rightarrow n'_{max} = \sqrt{\frac{\Delta E_c'}{E_{1c}^{\infty}}} = \sqrt{2}\, n_{max} = 2.3$

$\rightarrow$ there will be 3 confined state.

18.

20 wells, because of the energy states splitting. (Assuming the wave function of the first well can still affect the 10^{th} well.)

19.

Since the electrons in InGaAsP material has quite high mobility and they tend to escape from the quantum at high temperature, decreasing the injection efficiency. To improve the performance of laser at higher temperature, the electron leakage current can be decreased by increasing the conduction band offset.

20. Blue laser has lower wavelength than the red laser – thus, the minimum spot size that can be resolved on a Blu ray disc is smaller than that of a regular DVD. To get an even higher capacity, we should use an ultra violet laser.

21. Aixtron and Veeco.

22. Determine the percentage of Al and In in the AlInAs layers to lattice match to InP.

Solution: Assume that the percentage is x, that is $Al_xIn_{1-x}As$

The lattice constant of InP, AlAs and InAs are

$a_{InP} = 5.8688$ Å, $a_{AlAs} = 5.660$ Å, $a_{InAs} = 6.0583$ Å respectively.

Using Vegard's law,

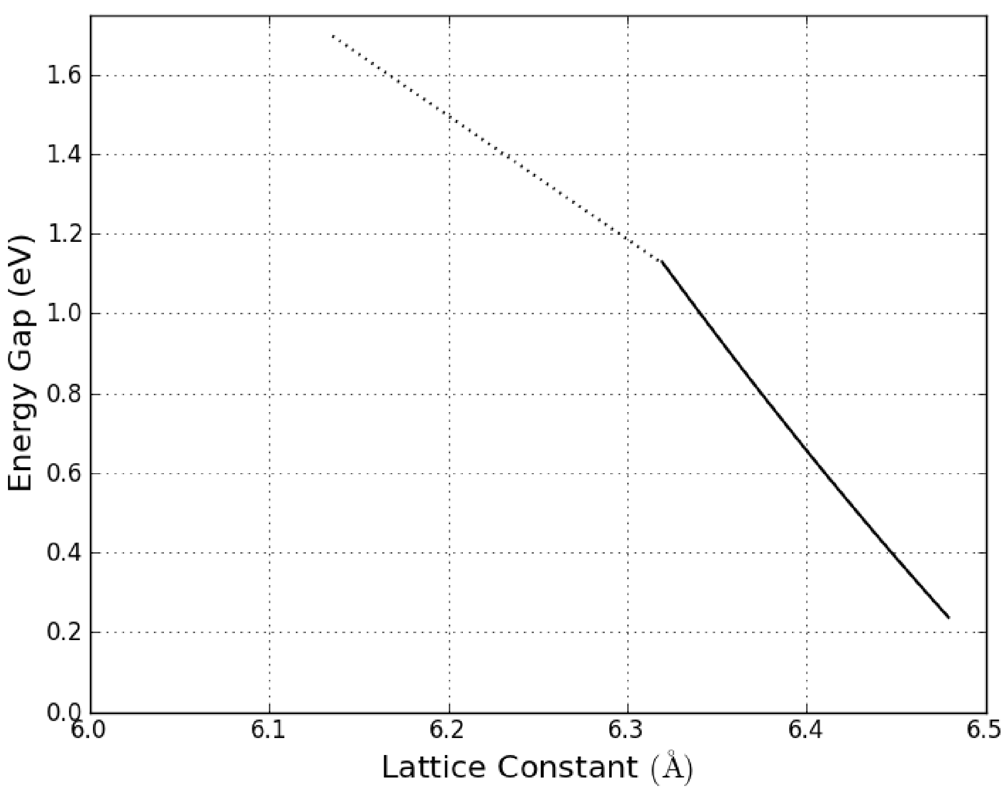
$$a_{InP} = x\, a_{AlAs} + (1-x)\, a_{InAs}$$

so

$$x = \frac{a_{InP} - a_{InAs}}{a_{AlAs} - a_{InAs}} = \frac{5.8688-6.0583}{5.660-6.0583} = 0.48$$

that is the percentage of Al should be 48% and In be 52%

23. When interpolating, one must be careful if different bands come into play in the process as discussed in the text. For this case, InAlSb's bandgap changes from direct to indirect for x = 0.535. The plot is shown below.



1.24

a)

$$E = E_1 + 2\Delta E \cos(ka) \qquad (A1.27)$$

$$\Delta E = 0.2 \text{ eV}$$

$$a = 0.3 \text{ nm}$$

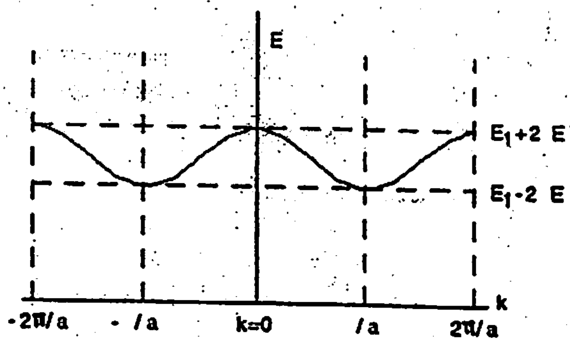


Figure 1.6a. Energy band over the first two Brillouin zones ($\frac{-2\pi}{a} \le k \le \frac{+2\pi}{a}$)

---

b)

$$m^* = \hbar^2 \left(\frac{\delta^2 E}{\delta k^2}\right)^{-1}$$

$$= \hbar \left(\frac{\delta}{\delta k}\frac{\delta}{\delta k}(E_1 + 2\Delta E \cos(ka))\right)^{-1}$$

$$= \frac{-\hbar^2}{2\Delta E a^2 \cos(ka)}$$

For a minimum of the energy band, $\cos(ka) = -1 \quad m^* > 0$.

For a maximum of the energy band, $\cos(ka) = +1 \quad m^* < 0$.

$$\frac{m^*}{m_0} = \pm \frac{\hbar^2}{2\Delta E a^2 m_0}$$

$$= \pm \frac{(6.59 \times 10^{-16} \text{ eV s})^2 \, (3 \times 10^{10} \frac{\text{cm}}{\text{s}})^2}{2(0.2 \text{ eV})(0.3 \times 10^{-7} \text{ cm})^2 \, (0.511 \times 10^6 \text{ eV})}$$

$$= \pm\, 2.12$$

where we have used $m_0 = 0.511 \text{ meV}/c^2$.

1.25

$$E_g(GaAs) \approx 1.42 \text{ eV}$$
$$\lambda_g = \frac{1.24 \text{ eV}\mu\text{m}}{E_g}$$
$$= 873 \text{ nm}$$

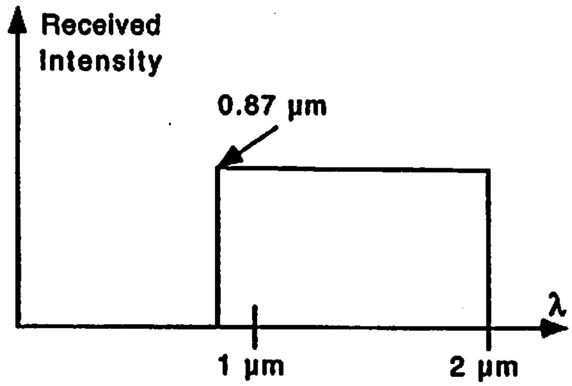


Figure 1.7. Recieved intensity vs. wavelength

Absorption: (R_{21})

All photons with energies greater than the bandgap (i.e. wavelengths < 873 nm) will be absorbed (This assumes reasonably low powers and that the wafer is more than a few microns thick.) This process will generate electron-hole pairs in the sample.

Non-radiative Recombination: (R_{nr})

The electron-hole pairs must recombine and one way is non-radiatively. The e-h pair will recombine and dissipate energy as lattice vibration (heat).

Spontaneous and Stimulated Emission: (R_{sp} and R_{st})

Some e-h pairs will recombine and create a photon. Photons generated by both processes will be near the bandgap energy (with a thermal distribution). Measuring the peak wavelength of the light emitted from a semiconductor when it is illuminated by an above bandgap source is a good way to determine the bandgap of an unknown material.

1.26

a) The semiconductor is being "bleached" by the Argon pump. The Argon laser light is being absorbed and this is generating e-h pairs. If the e-h pairs are being generated at a fast enough rate, the number of available states in the conduction band and valence band is being reduced so there aren't enough to completely absorb the GaAs laser light.

---

b) If the power is doubled to 6 W, the rate which e-h hole pairs are generated is not doubled: If the GaAs power is only 50% absorbed, additional power from the Argon laser will be less than fully absorbed. So 6 W will not double the e-h generation rate. Also, the recombination rate increases if more e-h pairs are present because $R_{sp} \propto np$. So to completely bleach the material, more power is needed to overcome both the decreased absorption and the increased recombination rate.

1.27

T = 125 °C
5kT = 173 meV
n = p = 4*10^18 cm^-3

As given in the appendix 2, expression (A2.7), the carrier density for a quantum well material is given by

$$N = \frac{k \cdot T \cdot m^*}{\pi \cdot \hbar^2 \cdot d} \sum_{n_z} \ln\left(1 + e^{\frac{E_F - E_{nz}}{k \cdot T}}\right)$$

Based on the given data, we can solve for

$$\sum_{n_z} \ln\left(1 + e^{\frac{E_F - E_{nz}}{k \cdot T}}\right) = \frac{\pi \cdot \hbar^2 \cdot d}{k \cdot T \cdot m^*} n = \frac{\pi \cdot (6.59\times10^{-16} eVs)^2 (80\times10^{-8} cm)(4\times10^{18} cm^{-3})}{0.034(0.067) \cdot \frac{0.511\times10^6 eV}{3\times10^{10} cm/s}} = 3.38$$

Now, we need to solve for E_F, and the quantum well energies E_{ni}, such that the left side of the equation is equal to the right side. This will be an iterative process, in which we choose a value for V_0, then find the energies of all possible quantum states, and use the relation between V_0 and E_F, $V_0 = 5kT + E_F$.

Solve for the quantum well energy levels, E_i based on problem 1.3(b). Note that this calculation is independent on the value of V_0, that is, V_0 will only determine the number of energy states in the well. After performing several iterations, we conclude that there will be only two states in the well, and for V_0= 323.6meV, we get

$$E_1^{\infty} = 87.68\text{ meV} \rightarrow n_{max} = 1.92 \rightarrow n_{qw1} = 0.75, \quad n_{qw2} = 1.45$$

E_1 = 49.32 meV
E_2 = 184.47 meV
E_F = 150.6 meV
Plugging in these values, we get

$$\sum_{n_z} \ln\left(1 + e^{\frac{E_F - E_{nz}}{k \cdot T}}\right) = \frac{\pi \cdot \hbar^2 \cdot d}{k \cdot T \cdot m^*} n = 3.38$$

Assume that 2/3 of the bandgap discontinuity occurs in the conduction band,

$$V_0 = \Delta E_C = \frac{2}{3}\Delta E_g \Rightarrow \Delta E_g = 485.4\text{ meV}$$

We will assume that the bandgaps change by the same amount when heated from 300K to 400K. Then we can use the bandgap values for 300K. Use linear extrapolation between GaAs and $Al_{0.2}Ga_{0.8}As$:

$$\Delta E_g = 0.485\text{eV} = \frac{x}{0.2}(1.673\text{ eV} - 1.424\text{ eV}) \rightarrow x = 0.39 \text{ or } 39\%\text{ Al}$$

1.28

a) The expression

$$E_F - E_C = kT \ln\left(\frac{N}{N_c}\right)$$

is not applicable as E_F approaches ∞ (or is much greater than the conduction band). A correction can be made by adding terms of the Joyce-Dixon approxmiation:

$$E_F - E_C = kT\left[\ln\left(\frac{N}{N_c}\right) + \sum_n A_n \left(\frac{N}{N_c}\right)^n\right] \qquad \text{(Table } A2.2\text{)}$$

$A_1 = +3.54 \times 10^{-1}$
$A_2 = -4.95 \times 10^{-3}$
$A_3 = +1.48 \times 10^{-4}$
$A_4 = -4.43 \times 10^{-6}$

Using the first of the four terms, we can get a reasonable approximation:

$$N_c = 2\left(\frac{2\pi m_C^* kT}{h^2}\right)^{3/2} = 2.54 \times 10^{19}\ m_c^{*3/2} \qquad (A2.10)$$

At 300 K, from Table 1.1,
For GaAs: $m_C^* = 0.067 \quad N_C = 4.40 \times 10^{17}\ \text{cm}^{-3}$
For InGaAsP(1.3μm): $m_C^* = 0.056 \quad N_C = 3.36 \times 10^{17}\ \text{cm}^{-3}$

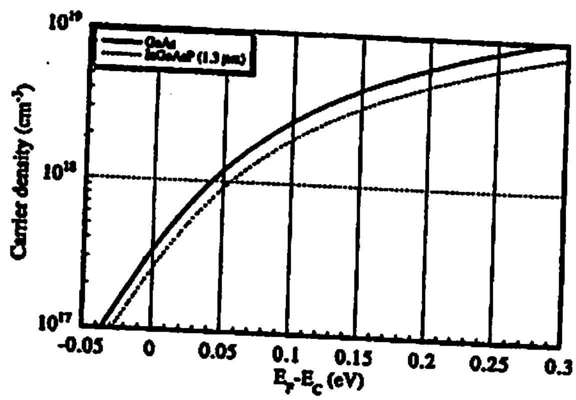


Figure 1.10a. Recieved intensity vs. wavelength

b) At 300 K, $N = 4 \times 10^{18}\ \text{cm}^{-3}$.

The problem doesn't specify what temperature to use for the $5kT$ measurement. Since carrier density is measured at 300K, it makes sense to use $T = 300K$, which implies

$5kT = 130$ meV

From part (a),

$E_F - E_C = 142$ meV

So

$$V_0 \equiv (E_F - E_C) + 5kT = 272\ \text{meV}$$

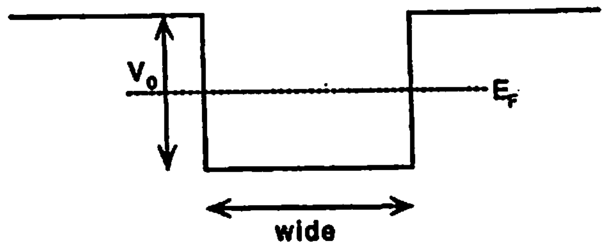


Figure 1.10b. Conduction band structure

SCH barrier material is found using linear extrapolation from the energy gap values given for $GaAs$ and $Al_{0.2}Ga_{0.8}As$ in Table 1.1. (Note: $AlAs$ is not used for extrapolation because it is not a direct bandgap.)

$$\frac{2}{3}\Delta E_g = 0.272 \text{ eV} = \frac{2}{3}\,\frac{x}{0.2}(1.673 \text{ eV} - 1.424 \text{ eV}) \rightarrow \quad x = 0.328$$

So, to satisfy the $5kT$ requirement, we require an alloy with at least 32.8 % aluminum.

$Al_{.33}Ga_{.67}As$

1.29

For a quantum wire with $dz \gg 10$ nm,

$$N_s(n_z) = 2n_z$$

$$E_z = \frac{\hbar^2\pi^2}{2m^*}\frac{n_z^2}{d_z^2}$$

Solve for n_z:

$$n_z = \sqrt{\frac{2m^* d_z^2 E_z}{\hbar^2\pi^2}}$$

$$N_s(E_z) = 2\sqrt{\frac{2m^* d_z^2}{\hbar^2\pi^2}}\sqrt{E_z}$$

$$\rho(E) = \frac{dN_s}{dE_z} = \sqrt{\frac{2m^* d_z^2}{\hbar^2\pi^2}}\frac{1}{dx\,dy\,dz}\frac{1}{\sqrt{E_z}} \qquad \text{for each level}$$

$$\begin{aligned}\rho(E) &= \frac{1}{dx\,dy}\sum_{nx}\sum_{ny}\frac{dN_s}{dE_z} \\ &= \sum_{nx}\sum_{ny}\sqrt{\frac{2m^*}{\hbar^2\pi^2}}\frac{1}{\sqrt{E-E_{nx,ny}}}H(E-E_{nx,ny})\end{aligned}$$

where $H(E-E_{nx,ny})$ is the Heavyside step function.

1.30

Use rectangular coordinates with the momentum variable and follow examples given in Appendix I, pp. 405-410.

$$\rho(p_x, p_y, p_z) = 2\left(\frac{2}{h}\right)^3 dp_x dp_y dp_z \qquad (A1.42)$$

$$dp_x dp_y dp_z = \sum_{p_x} \frac{2\pi\sqrt{p^2 - p_x^2}dp}{4} H(p - p_x)$$

The factor of 4 in the denominator accounts for the fact that p can be positive or negative.

$p_{nx} = \frac{\hbar \pi n_x}{d_x}$

$$H(p - p_x) = \begin{cases} 1 & p \geq p_x \\ 0 & p < p_x \end{cases}$$

$$\rho(p_x, p_y, p_z) = \frac{8\pi}{h^3} \sum_{n_x} \sqrt{p^2 - p_{n_x}^2}\, H(p - p_x)$$

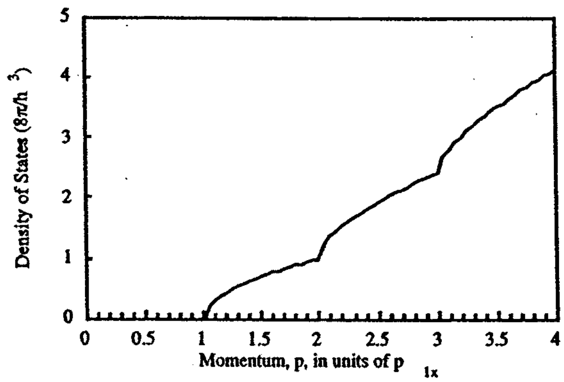


Figure 1.12. Density of states vs. momentum

1.31

$$\nabla^2 E = \mu\epsilon \frac{\delta^2 E}{\delta t^2} \tag{A3.1}$$

$$E(x,y,z,t) = \hat{e}_i E_0 U(x,y)\ e^{j(\omega t - \tilde{\beta} z)} \tag{A3.2}$$

Evaluating the first equation above using the description of E given by the second equation, we have
Left hand side:

$$\begin{aligned}\nabla^2 E &= \hat{e}_i E_0 \nabla_T^2 U(x,y)\ e^{j(\omega t - \tilde{\beta} z)} + (-\tilde{\beta}^2)\hat{e}_i E_0 U(x,y)\ e^{j(\omega t - \tilde{\beta} z)} \\ &= \left(\frac{\nabla^2 U(x,y)}{U(x,y)} - \tilde{\beta}^2\right) E\end{aligned}$$

Right hand side:

$$\mu\epsilon \frac{\delta^2 E}{\delta t^2} = -\omega^2 \mu\epsilon E$$

Thus, Eq. A3.1 can be rewritten as

$$\left(\frac{\nabla^2 U(x,y)}{U(x,y)} - \tilde{\beta}^2\right) E = -\omega^2 \mu\epsilon$$

Assume that $\mu = \mu_0$.
$k_0^2 = \omega^2 \mu_0 \epsilon_0$ and $n^2 = \frac{\epsilon}{\epsilon_0}$. Rearranging the above equation and dividing by E (for $E \neq 0$), we get

$$0 = \left(k_0^2 n^2 - \tilde{\beta}^2\right) U(x,y) + \nabla^2 U(x,y) \quad \text{for } E \neq 0$$

By continuity of E, this equation is also true for $E = 0$.

1.32

See Table 1.1 for the indicies of refraction for the materials in the waveguide:

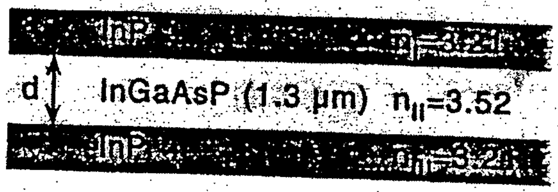


Figure 1.14. InGaAsP waveguide with $d = 0.2\mu m$

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a) Using normalized parameters from Appendix 3, we have

$$\begin{aligned} V &= k_0 d\sqrt{n_{II}^2 - n_{III}^2} \\ &= \frac{2\pi}{1.3\mu m}\, 0.2\mu m\, \sqrt{3.52^2 - 3.21^2} \qquad (A3.12) \\ &= 1.40 \end{aligned}$$

Since $V < \pi$, the guide is single-mode (see Figure A3.2).

---

b) First, find $\overline{n}$:

Method 1:

Again using normalized parameters, we have for a symmetric guide:

$$a = 0$$

Using Fig. A3.2, we determine

$$b \approx 1 - \frac{\ln(1+V^2/2)}{V^2/2} = 0.303$$

Using b, we can find $\overline{n}$:

$$b = \frac{\overline{n}^2 - n_{III}^2}{n_{II}^2 - n_{III}^2} \qquad (A3.12)$$

$$\overline{n} = \sqrt{0.37(3.52^2 - 3.21^2) + 3.21^2} = 3.307$$

$\beta = \frac{2\pi\overline{n}}{\lambda}$. k_x and γ_x can then be found by Eq. A3.7.

Method 2 (more accurate, but more time consuming):

$\overline{n}$ can also be found by solving the characteristic equation.

$$k_x \tan\left(\frac{k_x d}{2}\right) = \gamma \qquad (A3.9)$$

$$\sqrt{k_0^2 n_{II}^2 - \beta^2}\, \tan\left(\frac{d}{2}\sqrt{k_0^2 n_{II}^2 - \beta^2}\right) = \sqrt{\beta^2 - k_0 n_{III}^2}$$

$$\sqrt{\left(\frac{2\pi}{1.3\mu m}\right)^2 3.52^2 - \beta^2}\, \tan\left(\frac{0.2\mu m}{2}\sqrt{\left(\frac{2\pi}{1.3\mu m}\right)^2 3.52^2 - \beta^2}\right) = \sqrt{\beta^2 - \left(\frac{2\pi}{1.3\mu m}\right) 3.21^2}$$

Solve iteratively to find

$$\beta = 15.98\mu\text{m}^{-1} \qquad (\rightarrow \bar{n} = 3.306)$$

$$k_x = 5.84\mu\text{m}^{-1}$$

$$\gamma = 3.83\mu\text{m}^{-1}$$

The electric field of the mode is then given by

$$U_{II} = A\cos(k_x x)$$

$$U_{I,III} = B\ e^{-\gamma|x|}$$

Use boundary conditions to solve for the coefficients A and B. The electric field must be continuous to satisfy Maxwell's equations. Therefore,

$$U_{II}(x = 0.1) = U_{III}(x = 0.1)$$

$$A\cos((5.84)(0.1)) = B\ e^{-(3.83)(0.1)}$$

$$B = 1.22A$$

So

$$U_{II} = A\cos((5.84\mu\text{m}^{-1})x)$$

$$U_{I,III} = 1.22A\ e^{-(3.83\mu\text{m}^{-1})|x|}$$

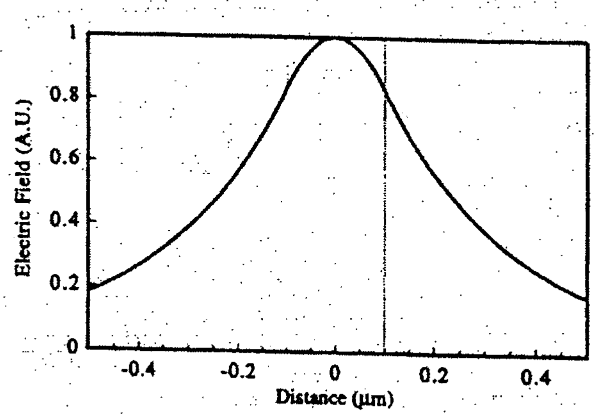

Figure 1.14b. Waveguide electric field profile

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c)

$$E^2(x = 0) = (A \quad \cos(0))^2 = A^2$$

$$E^2(x = 0.6\mu\text{m}) = (1.22Ae^{-(3.83)(0.6)})^2 = 0.015\,A^2$$

$$\frac{E^2(x = 0.6\mu\text{m})}{E^2(x = 0)} = 1.5\%$$

Note that energy density is really $\frac{\epsilon}{2}E^2$. We ignored the variation in index ($n = \sqrt{\epsilon}$) across the waveguide.

$$\Rightarrow \text{ratio} = 0.015\ \frac{\varepsilon_{core}}{\varepsilon_{cladding}} = 1.8\%$$

d)

$$\beta = \frac{2\pi\overline{n}}{\lambda}$$

$$n_{eff} = \overline{n} = \frac{\lambda\beta}{2\pi} = \frac{(1.3\mu\text{m})(15.98\mu\text{m}^{-1})}{2\pi} = 3.31$$

e) Using equation A3.14 or A3.15, we can solve for the transverse confinement factor:

$$\begin{aligned}\Gamma &= \frac{1 + \frac{2\gamma d}{V^2}}{1 + \frac{2}{\gamma d}} \\ &= \frac{1 + \frac{2(3.83)(0.2)}{1.40^2}}{1 + \frac{2}{(3.83)(0.2)}} \\ &= 49.3\%\end{aligned}$$

$$\begin{aligned}\Gamma &\approx \frac{V^2}{2 + V^2} \\ &\approx \frac{(1.40)^2}{2 + (1.40)^2} \\ &\approx 49.5\%\end{aligned}$$

$$\Gamma = 49.3\%$$

1.33

Using the result from Problem 1.14(d), the central region can be represented as a material with an index of $\bar{n} = 3.31$.

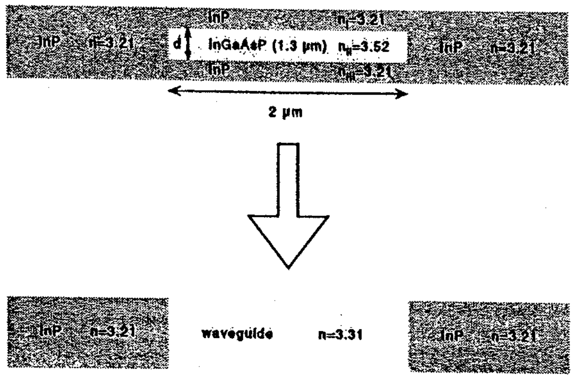

Figure 1.15. Effective index method is used to replace central region

The rest of the problem is solved with a method similar that used for Problem 1.14.

---

a)

$$V = k_0 d \sqrt{n_{II}^2 - n_{III}^2} = 7.80$$

$$a = 0$$

From Fig A3.2:

$$b \approx 0.91$$

$$n_{eff} = \bar{n} = \sqrt{b(n_{II}^2 - n_{III}^2) + n_{III}^2} = 3.30$$

---

b) Figure A3.2 shows that there are 3 possible modes.

---

c) Equation A3.15 works well for small index differences:

$$\Gamma_x = \frac{V^2}{2 + V^2} = \frac{(7.80)^2}{2 + (7.80)^2} = 96.8\%$$

1.34

$$U(x,y) = U\sin(k_x x)\ \sin(k_y y) \qquad (A3.17)$$

Since the mode is strongly confined, we can approximate the boundary conditions by letting the electric field be zero at the edge of the pillar.

$$U(x,y) = 0 \qquad x > 5\mu\text{m or } y > 5\mu\text{m}$$

$k_x = \frac{m\pi}{d}$ where $m = 1,2,3,\ldots$
$k_y = \frac{n\pi}{d}$ where $n = 1,2,3,\ldots$
$k_z = \beta = constant$

$$k_{mn}^2 = k_x^2 + k_y^2 + \beta^2$$

For the fundamental mode, $m = n = 1$.

$$\lambda_{11} = \frac{2\pi}{k_{11}} = 1.00\mu\text{m}$$

$$\left(\frac{2\pi(3.30)}{1.0\mu\text{m}}\right)^2 = \left(\frac{\pi}{5\mu\text{m}}\right)^2 + \left(\frac{\pi}{5\mu\text{m}}\right)^2 + \beta^2$$

Solve for β^2 to find

$$\beta^2 = 429.13\mu\text{m}^{-2}$$

Now, the mode spectrum can be written analytically as

$$\lambda_{mn} = \frac{2\pi(3.30)}{\sqrt{429.13\mu\text{m}^{-2} + n^2\left(\frac{\pi}{5\mu\text{m}}\right)^2 + m^2\left(\frac{\pi}{5\mu\text{m}}\right)^2}}$$

The first six modes are

$$\lambda_{11} = 1.00\mu\text{m}$$
$$\lambda_{21} = \lambda_{12} = 0.9986\mu\text{m}$$
$$\lambda_{22} = 0.9972\mu\text{m}$$
$$\lambda_{13} = \lambda_{31} = 0.9963\mu\text{m}$$

other modes have shorter wavelengths:

$$\lambda_{23} = \lambda_{32} = 0.9950\mu\text{m}$$
$$\lambda_{33} = 0.9927\mu\text{m}$$

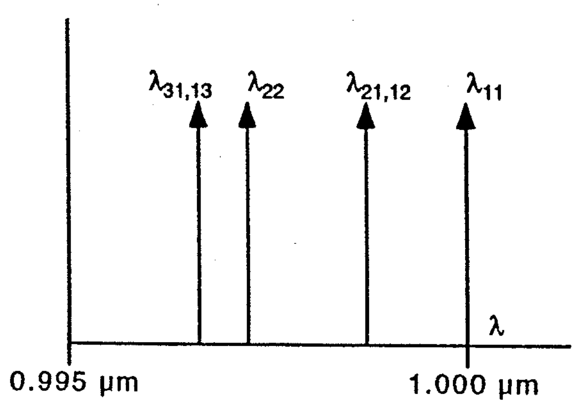

Figure 1.16. Mode spectrum including first six lateral modes.

1.35

The electric field for the fundamental mode is given by Eqs. A3.5 and A3.6 in combination with Eq. A3.8:

$$U(x) = \begin{cases} A\cos(k_x x) & |x| < d \\ Acos(kxd/2)\ e^{\gamma d/2}\ e^{-\gamma|x|} & |x| \geq d \end{cases}$$

The confinement factor is defined by

$$\Gamma_x = \frac{\int_{-d/2}^{d/2} |U(x)|^2 dx}{\int_{-\infty}^{\infty} |U(x)|^2 dx} \qquad (A3.13)$$

Expand the numerator

$$\begin{aligned}
\int_{-d/2}^{d/2} |U(x)|^2 dx &= \int_{-d/2}^{d/2} |A|^2 \cos^2(k_x x) dx \\
&= |A|^2 \int_{-d/2}^{d/2} \frac{1}{2}(1 - \sin^2(k_x x) + \cos^2(k_x x)) dx \\
&= |A|^2 \int_{-d/2}^{d/2} \frac{1}{2}(1 + \cos(2k_x x)) dx \\
&= \frac{1}{2}|A|^2 \left[x + \frac{1}{2k_x} \sin(2k_x x)) \right|_{-d/2}^{d/2} \\
&= \frac{1}{2}|A|^2 \left(d + \frac{2}{2k_x} \sin(k_x d)) \right)
\end{aligned}$$

Expand the denominator

$$\begin{aligned}
\int_{-\infty}^{\infty} |U(x)|^2 dx &= \int_{-d/2}^{d/2} |U(x)|^2 dx + 2\int_{d/2}^{\infty} |U(x)|^2 dx \\
&= \frac{1}{2}|A|^2 \left(d + \frac{2}{2k_x} \sin(k_x d)) \right) + 2\int_{d/2}^{\infty} |A|^2 cos^2(k_x d/2)\ e^{2\gamma d/2}\ e^{-2\gamma|x|} dx \\
&= \frac{1}{2}|A|^2 \left(d + \frac{2}{2k_x} \sin(k_x d)) \right) + 2|A|^2 cos^2(k_x d/2)\ e^{\gamma d} \int_{d/2}^{\infty} e^{-2\gamma|x|} dx \\
&= \frac{1}{2}|A|^2 \left(d + \frac{2}{2k_x} \sin(k_x d)) \right) + 2|A|^2 cos^2(k_x d/2)\ e^{\gamma d} \left[\frac{-1}{2\gamma}\ e^{-2\gamma|x|} \right|_{d/2}^{\infty} \\
&= \frac{1}{2}|A|^2 \left(d + \frac{2}{2k_x} \sin(k_x d)) \right) + 2|A|^2 cos^2(k_x d/2)\ e^{\gamma d} \frac{-1}{2\gamma} \left(-\ e^{-2\gamma d/2} \right) \\
&= \frac{1}{2}|A|^2 \left(d + \frac{2}{2k_x} \sin(k_x d)) \right) + |A|^2 cos^2(k_x d/2) \frac{1}{\gamma}
\end{aligned}$$

So, we have

$$\Gamma_x = \frac{\left(1 + \frac{1}{k_x d} \sin(k_x d))\right)}{\left(1 + \frac{1}{k_x d} \sin(k_x d))\right) + \frac{2}{\gamma d} cos^2(k_x d/2)}$$

Use trigonometric identities with Eqs. A3.9 and A3.7 to write

$$\cos^2\left(\frac{k_x d}{2}\right) = \frac{1}{1+\tan^2\left(\frac{k_x d}{2}\right)}$$
$$= \frac{1}{1+\left(\frac{\gamma d}{k_x d}\right)^2}$$
$$= \frac{(k_x d)^2}{(k_x d)^2 + (\gamma d)^2}$$
$$= \frac{(k_x d)^2}{V^2}$$

$$\sin(k_x d) = 2\cos\left(\frac{k_x d}{2}\right)\sin\left(\frac{k_x d}{2}\right)$$
$$= 2\cos^2\left(\frac{k_x d}{2}\right)\tan\left(\frac{k_x d}{2}\right)$$
$$= 2\cos^2\left(\frac{k_x d}{2}\right)\tan\left(\frac{k_x d}{2}\right)$$
$$= 2\frac{(k_x d)^2}{V^2}\frac{\gamma d}{k_x d}$$
$$= \frac{2(k_x d)(\gamma d)}{V^2}$$

Using these substitutions, we can complete the derivation of Eq. A3.14:

$$\Gamma_x = \frac{\left(1+\frac{1}{k_x d}\sin(k_x d)\right)}{\left(1+\frac{1}{k_x d}\sin(k_x d)\right)+\frac{2}{\gamma d}cos^2(k_x d/2)}$$
$$= \frac{\left(1+\frac{1}{k_x d}\left[\frac{2(k_x d)(\gamma d)}{V^2}\right]\right)}{\left(1+\frac{1}{k_x d}\left[\frac{2(k_x d)(\gamma d)}{V^2}\right]\right)+\frac{2}{\gamma d}\left[\frac{(k_x d)^2}{V^2}\right]}$$
$$= \frac{V^2+2(\gamma d)}{V^2+2(\gamma d)+\frac{2(k_x d)^2}{\gamma d}}$$
$$= \frac{V^2+2(\gamma d)}{V^2+2\frac{(\gamma d)^2+(k_x d)^2}{\gamma d}}$$
$$= \frac{V^2+2(\gamma d)}{V^2\left[1+\frac{2}{\gamma d}\right]}$$
$$= \frac{1+\frac{2(\gamma d)}{V^2}}{1+\frac{2}{\gamma d}}$$

$$\Gamma_x = \frac{1+2\gamma d/V^2}{1+2/\gamma d} \qquad (A3.14)$$

Verify Eq. A3.15:

Verify Eq. A3.15 by producing plot similar to A3.3:

For $\lambda = 1.0\mu$m, $n_I = n_{III} = 3.3$, and $n_2 = 3.6$,

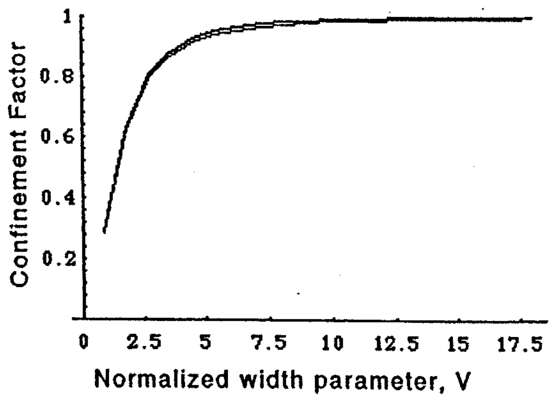

Figure 1.17. Confinement factor vs. V as waveguide width is varied.

1.36

Use the infinite barrier approximation for the lateral direction:

$$E_1^{\infty} = 3.76\frac{m_0}{m_e^*}\left(\frac{100\text{Å}}{d}\right)^2 \text{ meV} \tag{A1.14}$$

From Table 1.1 for GaAs, $m_e^* = 0.067m_0$.

$$\Delta E_1^{\infty} = 10 \text{ meV} = 3.76\frac{1}{0.067}\left(\frac{100\text{Å}}{d}\right)^2 \text{ meV}$$

Solve for d:

$$d = \sqrt{\frac{3.76}{(10)(0.067)}}\,(100\text{Å}) = 237\text{Å}$$