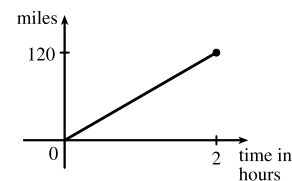


## 1 FUNCTIONS AND MODELS

### 1.1 Four Ways to Represent a Function

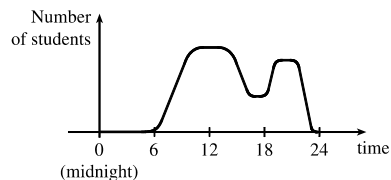
- The functions  $f(x) = x + \sqrt{2-x}$  and  $g(u) = u + \sqrt{2-u}$  give exactly the same output values for every input value, so  $f$  and  $g$  are equal.
- $f(x) = \frac{x^2 - x}{x - 1} = \frac{x(x - 1)}{x - 1} = x$  for  $x - 1 \neq 0$ , so  $f$  and  $g$  [where  $g(x) = x$ ] are not equal because  $f(1)$  is undefined and  $g(1) = 1$ .
- The point  $(1, 3)$  is on the graph of  $f$ , so  $f(1) = 3$ .
  - When  $x = -1$ ,  $y$  is about  $-0.2$ , so  $f(-1) \approx -0.2$ .
  - $f(x) = 1$  is equivalent to  $y = 1$ . When  $y = 1$ , we have  $x = 0$  and  $x = 3$ .
  - A reasonable estimate for  $x$  when  $y = 0$  is  $x = -0.8$ .
  - The domain of  $f$  consists of all  $x$ -values on the graph of  $f$ . For this function, the domain is  $-2 \leq x \leq 4$ , or  $[-2, 4]$ .  
The range of  $f$  consists of all  $y$ -values on the graph of  $f$ . For this function, the range is  $-1 \leq y \leq 3$ , or  $[-1, 3]$ .
  - As  $x$  increases from  $-2$  to  $1$ ,  $y$  increases from  $-1$  to  $3$ . Thus,  $f$  is increasing on the interval  $[-2, 1]$ .
- The point  $(-4, -2)$  is on the graph of  $f$ , so  $f(-4) = -2$ . The point  $(3, 4)$  is on the graph of  $g$ , so  $g(3) = 4$ .
  - We are looking for the values of  $x$  for which the  $y$ -values are equal. The  $y$ -values for  $f$  and  $g$  are equal at the points  $(-2, 1)$  and  $(2, 2)$ , so the desired values of  $x$  are  $-2$  and  $2$ .
  - $f(x) = -1$  is equivalent to  $y = -1$ . When  $y = -1$ , we have  $x = -3$  and  $x = 4$ .
  - As  $x$  increases from  $0$  to  $4$ ,  $y$  decreases from  $3$  to  $-1$ . Thus,  $f$  is decreasing on the interval  $[0, 4]$ .
  - The domain of  $f$  consists of all  $x$ -values on the graph of  $f$ . For this function, the domain is  $-4 \leq x \leq 4$ , or  $[-4, 4]$ .  
The range of  $f$  consists of all  $y$ -values on the graph of  $f$ . For this function, the range is  $-2 \leq y \leq 3$ , or  $[-2, 3]$ .
  - The domain of  $g$  is  $[-4, 3]$  and the range is  $[0.5, 4]$ .
- From Figure 1 in the text, the lowest point occurs at about  $(t, a) = (12, -85)$ . The highest point occurs at about  $(17, 115)$ . Thus, the range of the vertical ground acceleration is  $-85 \leq a \leq 115$ . Written in interval notation, we get  $[-85, 115]$ .
- Example 1:* A car is driven at 60 mi/h for 2 hours. The distance  $d$  traveled by the car is a function of the time  $t$ . The domain of the function is  $\{t \mid 0 \leq t \leq 2\}$ , where  $t$  is measured in hours. The range of the function is  $\{d \mid 0 \leq d \leq 120\}$ , where  $d$  is measured in miles.



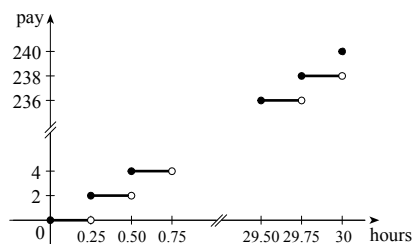
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## 10 □ CHAPTER 1 FUNCTIONS AND MODELS

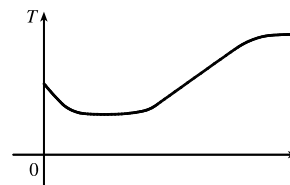
*Example 2:* At a certain university, the number of students  $N$  on campus at any time on a particular day is a function of the time  $t$  after midnight. The domain of the function is  $\{t \mid 0 \leq t \leq 24\}$ , where  $t$  is measured in hours. The range of the function is  $\{N \mid 0 \leq N \leq k\}$ , where  $N$  is an integer and  $k$  is the largest number of students on campus at once.



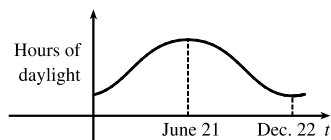
*Example 3:* A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay  $P$  is a function of the number of hours worked  $h$ . The domain of the function is  $[0, 30]$  and the range of the function is  $\{0, 2.00, 4.00, \dots, 238.00, 240.00\}$ .



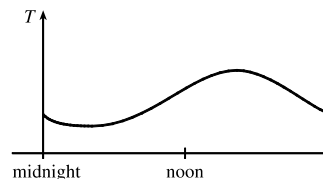
7. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.
8. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is  $[-2, 2]$  and the range is  $[-1, 2]$ .
9. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is  $[-3, 2]$  and the range is  $[-3, -2) \cup [-1, 3]$ .
10. No, the curve is not the graph of a function since for  $x = 0, \pm 1$ , and  $\pm 2$ , there are infinitely many points on the curve.
11. (a) When  $t = 1950$ ,  $T \approx 13.8^\circ\text{C}$ , so the global average temperature in 1950 was about  $13.8^\circ\text{C}$ .  
 (b) When  $T = 14.2^\circ\text{C}$ ,  $t \approx 1990$ .  
 (c) The global average temperature was smallest in 1910 (the year corresponding to the lowest point on the graph) and largest in 2005 (the year corresponding to the highest point on the graph).  
 (d) When  $t = 1910$ ,  $T \approx 13.5^\circ\text{C}$ , and when  $t = 2005$ ,  $T \approx 14.5^\circ\text{C}$ . Thus, the range of  $T$  is about  $[13.5, 14.5]$ .
12. (a) The ring width varies from near 0 mm to about 1.6 mm, so the range of the ring width function is approximately  $[0, 1.6]$ .  
 (b) According to the graph, the earth gradually cooled from 1550 to 1700, warmed into the late 1700s, cooled again into the late 1800s, and has been steadily warming since then. In the mid-19th century, there was variation that could have been associated with volcanic eruptions.
13. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



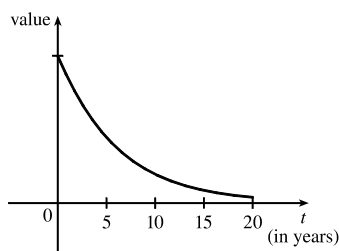
14. Runner A won the race, reaching the finish line at 100 meters in about 15 seconds, followed by runner B with a time of about 19 seconds, and then by runner C who finished in around 23 seconds. B initially led the race, followed by C, and then A. C then passed B to lead for a while. Then A passed first B, and then passed C to take the lead and finish first. Finally, B passed C to finish in second place. All three runners completed the race.
15. (a) The power consumption at 6 AM is 500 MW, which is obtained by reading the value of power  $P$  when  $t = 6$  from the graph. At 6 PM we read the value of  $P$  when  $t = 18$ , obtaining approximately 730 MW.
- (b) The minimum power consumption is determined by finding the time for the lowest point on the graph,  $t = 4$ , or 4 AM. The maximum power consumption corresponds to the highest point on the graph, which occurs just before  $t = 12$ , or right before noon. These times are reasonable, considering the power consumption schedules of most individuals and businesses.
16. The summer solstice (the longest day of the year) is around June 21, and the winter solstice (the shortest day) is around December 22. (Exchange the dates for the southern hemisphere.)



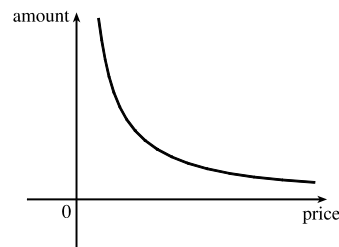
17. Of course, this graph depends strongly on the geographical location!



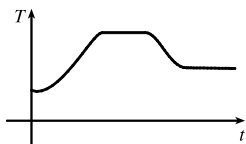
18. The value of the car decreases fairly rapidly initially, then somewhat less rapidly.



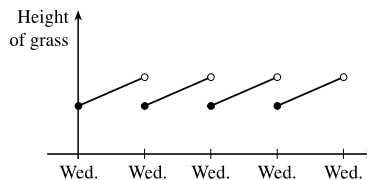
19. As the price increases, the amount sold decreases.



20. The temperature of the pie would increase rapidly, level off to oven temperature, decrease rapidly, and then level off to room temperature.

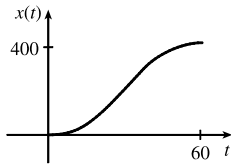


21. Height of grass vs time t. The graph shows a series of discrete points connected by line segments, indicating a step-like increase over time.

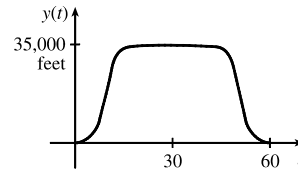


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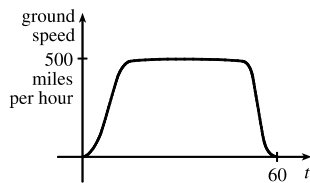
22. (a)



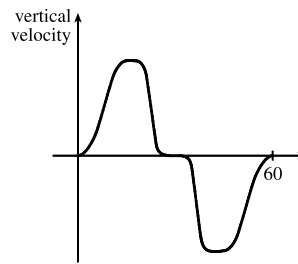
(b)



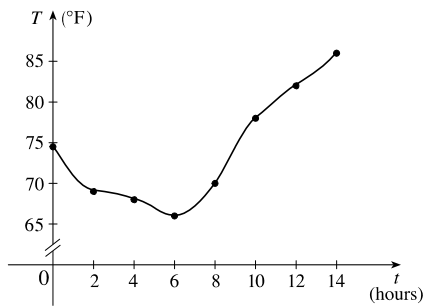
(c)



(d)

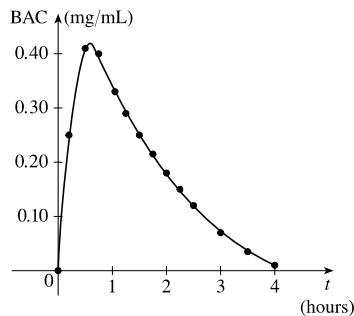


23. (a)



(b) 9:00 AM corresponds to  $t = 9$ . When  $t = 9$ , the temperature  $T$  is about  $74^\circ\text{F}$ .

24. (a)



(b) The blood alcohol concentration rises rapidly, then slowly decreases to near zero. Note that the BAC in this exercise is measured in mg/mL, not percent.

25.  $f(x) = 3x^2 - x + 2$ .

$$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12.$$

$$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16.$$

$$f(a) = 3a^2 - a + 2.$$

$$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2.$$

$$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a - 1 + 2 = 3a^2 + 5a + 4.$$

$$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4.$$

$$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2.$$

$$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2.$$

$$\begin{aligned} [f(a)]^2 &= [3a^2 - a + 2]^2 = (3a^2 - a + 2)(3a^2 - a + 2) \\ &= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4. \end{aligned}$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2.$$

26. A spherical balloon with radius  $r + 1$  has volume  $V(r + 1) = \frac{4}{3}\pi(r + 1)^3 = \frac{4}{3}\pi(r^3 + 3r^2 + 3r + 1)$ . We wish to find the amount of air needed to inflate the balloon from a radius of  $r$  to  $r + 1$ . Hence, we need to find the difference

$$V(r + 1) - V(r) = \frac{4}{3}\pi(r^3 + 3r^2 + 3r + 1) - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3r^2 + 3r + 1).$$

27.  $f(x) = 4 + 3x - x^2$ , so  $f(3+h) = 4 + 3(3+h) - (3+h)^2 = 4 + 9 + 3h - (9 + 6h + h^2) = 4 - 3h - h^2$ ,

$$\text{and } \frac{f(3+h) - f(3)}{h} = \frac{(4 - 3h - h^2) - 4}{h} = \frac{h(-3 - h)}{h} = -3 - h.$$

28.  $f(x) = x^3$ , so  $f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$ ,

$$\text{and } \frac{f(a+h) - f(a)}{h} = \frac{(a^3 + 3a^2h + 3ah^2 + h^3) - a^3}{h} = \frac{h(3a^2 + 3ah + h^2)}{h} = 3a^2 + 3ah + h^2.$$

$$29. \frac{f(x) - f(a)}{x - a} = \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{\frac{a - x}{xa}}{x - a} = \frac{a - x}{xa(x - a)} = \frac{-1(x - a)}{xa(x - a)} = -\frac{1}{ax}$$

$$\begin{aligned} 30. \frac{f(x) - f(1)}{x - 1} &= \frac{\frac{x+3}{x+1} - 2}{x - 1} = \frac{\frac{x+3 - 2(x+1)}{x+1}}{x - 1} = \frac{x+3 - 2x - 2}{(x+1)(x-1)} \\ &= \frac{-x+1}{(x+1)(x-1)} = \frac{-(x-1)}{(x+1)(x-1)} = -\frac{1}{x+1} \end{aligned}$$

31.  $f(x) = (x+4)/(x^2 - 9)$  is defined for all  $x$  except when  $0 = x^2 - 9 \Leftrightarrow 0 = (x+3)(x-3) \Leftrightarrow x = -3$  or  $3$ , so the domain is  $\{x \in \mathbb{R} \mid x \neq -3, 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ .

32.  $f(x) = (2x^3 - 5)/(x^2 + x - 6)$  is defined for all  $x$  except when  $0 = x^2 + x - 6 \Leftrightarrow 0 = (x+3)(x-2) \Leftrightarrow x = -3$  or  $2$ , so the domain is  $\{x \in \mathbb{R} \mid x \neq -3, 2\} = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ .

33.  $f(t) = \sqrt[3]{2t-1}$  is defined for all real numbers. In fact  $\sqrt[3]{p(t)}$ , where  $p(t)$  is a polynomial, is defined for all real numbers. Thus, the domain is  $\mathbb{R}$ , or  $(-\infty, \infty)$ .

34.  $g(t) = \sqrt{3-t} - \sqrt{2+t}$  is defined when  $3-t \geq 0 \Leftrightarrow t \leq 3$  and  $2+t \geq 0 \Leftrightarrow t \geq -2$ . Thus, the domain is  $-2 \leq t \leq 3$ , or  $[-2, 3]$ .

35.  $h(x) = 1/\sqrt[4]{x^2 - 5x}$  is defined when  $x^2 - 5x > 0 \Leftrightarrow x(x-5) > 0$ . Note that  $x^2 - 5x \neq 0$  since that would result in division by zero. The expression  $x(x-5)$  is positive if  $x < 0$  or  $x > 5$ . (See Appendix A for methods for solving inequalities.) Thus, the domain is  $(-\infty, 0) \cup (5, \infty)$ .

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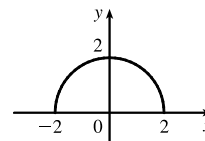
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36.  $f(u) = \frac{u+1}{1+\frac{1}{u+1}}$  is defined when  $u+1 \neq 0$  [ $u \neq -1$ ] and  $1 + \frac{1}{u+1} \neq 0$ . Since  $1 + \frac{1}{u+1} = 0 \Leftrightarrow$

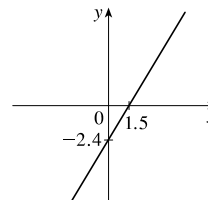
$$\frac{1}{u+1} = -1 \Leftrightarrow 1 = -u-1 \Leftrightarrow u = -2, \text{ the domain is } \{u \mid u \neq -2, u \neq -1\} = (-\infty, -2) \cup (-2, -1) \cup (-1, \infty).$$

37.  $F(p) = \sqrt{2-\sqrt{p}}$  is defined when  $p \geq 0$  and  $2-\sqrt{p} \geq 0$ . Since  $2-\sqrt{p} \geq 0 \Leftrightarrow 2 \geq \sqrt{p} \Leftrightarrow \sqrt{p} \leq 2 \Leftrightarrow$   
 $0 \leq p \leq 4$ , the domain is  $[0, 4]$ .

38.  $h(x) = \sqrt{4-x^2}$ . Now  $y = \sqrt{4-x^2} \Rightarrow y^2 = 4-x^2 \Leftrightarrow x^2 + y^2 = 4$ , so the graph is the top half of a circle of radius 2 with center at the origin. The domain is  $\{x \mid 4-x^2 \geq 0\} = \{x \mid 4 \geq x^2\} = \{x \mid 2 \geq |x|\} = [-2, 2]$ . From the graph, the range is  $0 \leq y \leq 2$ , or  $[0, 2]$ .

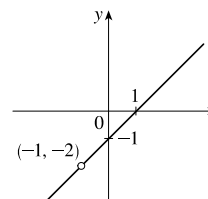


39. The domain of  $f(x) = 1.6x - 2.4$  is the set of all real numbers, denoted by  $\mathbb{R}$  or  $(-\infty, \infty)$ . The graph of  $f$  is a line with slope 1.6 and  $y$ -intercept  $-2.4$ .



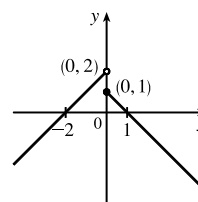
40. Note that  $g(t) = \frac{t^2-1}{t+1} = \frac{(t+1)(t-1)}{t+1} = t-1$  for  $t+1 \neq 0$ , i.e.,  $t \neq -1$ .

The domain of  $g$  is the set of all real numbers except  $-1$ . In interval notation, we have  $(-\infty, -1) \cup (-1, \infty)$ . The graph of  $g$  is a line with slope 1,  $y$ -intercept  $-1$ , and a hole at  $t = -1$ .



$$41. f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$$

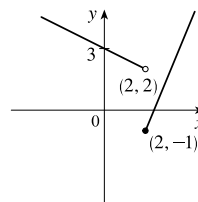
$$f(-3) = -3+2 = -1, f(0) = 1-0 = 1, \text{ and } f(2) = 1-2 = -1.$$



$$42. f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x < 2 \\ 2x - 5 & \text{if } x \geq 2 \end{cases}$$

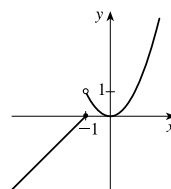
$$f(-3) = 3 - \frac{1}{2}(-3) = \frac{9}{2}, f(0) = 3 - \frac{1}{2}(0) = 3,$$

$$\text{and } f(2) = 2(2) - 5 = -1.$$



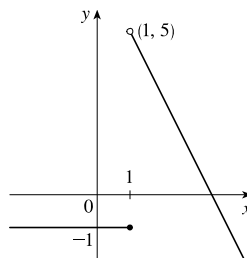
$$43. f(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

$$f(-3) = -3+1 = -2, f(0) = 0^2 = 0, \text{ and } f(2) = 2^2 = 4.$$



44.  $f(x) = \begin{cases} -1 & \text{if } x \leq 1 \\ 7 - 2x & \text{if } x > 1 \end{cases}$

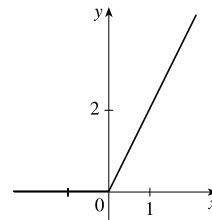
$f(-3) = -1, f(0) = -1, \text{ and } f(2) = 7 - 2(2) = 3.$



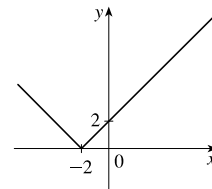
45.  $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

so  $f(x) = x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

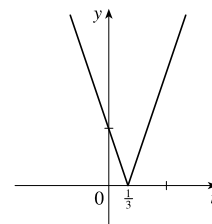
Graph the line  $y = 2x$  for  $x \geq 0$  and graph  $y = 0$  (the  $x$ -axis) for  $x < 0$ .



46.  $f(x) = |x + 2| = \begin{cases} x + 2 & \text{if } x + 2 \geq 0 \\ -(x + 2) & \text{if } x + 2 < 0 \end{cases}$   
 $= \begin{cases} x + 2 & \text{if } x \geq -2 \\ -x - 2 & \text{if } x < -2 \end{cases}$

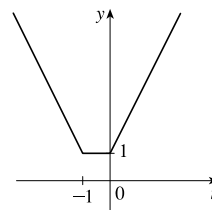


47.  $g(t) = |1 - 3t| = \begin{cases} 1 - 3t & \text{if } 1 - 3t \geq 0 \\ -(1 - 3t) & \text{if } 1 - 3t < 0 \end{cases}$   
 $= \begin{cases} 1 - 3t & \text{if } t \leq \frac{1}{3} \\ 3t - 1 & \text{if } t > \frac{1}{3} \end{cases}$



48.  $|t| = \begin{cases} t & \text{if } t \geq 0 \\ -t & \text{if } t < 0 \end{cases}$  and

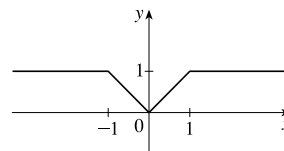
$|t + 1| = \begin{cases} t + 1 & \text{if } t + 1 \geq 0 \\ -(t + 1) & \text{if } t + 1 < 0 \end{cases} = \begin{cases} t + 1 & \text{if } t \geq -1 \\ -t - 1 & \text{if } t < -1 \end{cases}$



so  $h(t) = |t| + |t + 1| = \begin{cases} t + (t + 1) & \text{if } t \geq 0 \\ -t + (t + 1) & \text{if } -1 \leq t < 0 \\ -t + (-t - 1) & \text{if } t < -1 \end{cases} = \begin{cases} 2t + 1 & \text{if } t \geq 0 \\ 1 & \text{if } -1 \leq t < 0 \\ -2t - 1 & \text{if } t < -1 \end{cases}$

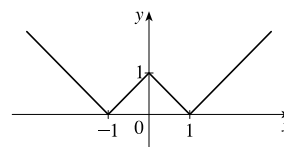
49. To graph  $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 1 & \text{if } |x| > 1 \end{cases}$ , graph  $y = |x|$  (Figure 16)

for  $-1 \leq x \leq 1$  and graph  $y = 1$  for  $x > 1$  and for  $x < -1$ .



We could rewrite  $f$  as  $f(x) = \begin{cases} 1 & \text{if } x < -1 \\ -x & \text{if } -1 \leq x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$ .

50.  $g(x) = \left| |x| - 1 \right| = \begin{cases} |x| - 1 & \text{if } |x| - 1 \geq 0 \\ -(|x| - 1) & \text{if } |x| - 1 < 0 \end{cases}$
- $$= \begin{cases} |x| - 1 & \text{if } |x| \geq 1 \\ -|x| + 1 & \text{if } |x| < 1 \end{cases}$$



$$= \begin{cases} x - 1 & \text{if } |x| \geq 1 \text{ and } x \geq 0 \\ -x - 1 & \text{if } |x| \geq 1 \text{ and } x < 0 \\ -x + 1 & \text{if } |x| < 1 \text{ and } x \geq 0 \\ -(-x) + 1 & \text{if } |x| < 1 \text{ and } x < 0 \end{cases} = \begin{cases} x - 1 & \text{if } x \geq 1 \\ -x - 1 & \text{if } x \leq -1 \\ -x + 1 & \text{if } 0 \leq x < 1 \\ x + 1 & \text{if } -1 < x < 0 \end{cases}$$

51. Recall that the slope  $m$  of a line between the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $m = \frac{y_2 - y_1}{x_2 - x_1}$  and an equation of the line connecting those two points is  $y - y_1 = m(x - x_1)$ . The slope of the line segment joining the points  $(1, -3)$  and  $(5, 7)$  is  $\frac{7 - (-3)}{5 - 1} = \frac{5}{2}$ , so an equation is  $y - (-3) = \frac{5}{2}(x - 1)$ . The function is  $f(x) = \frac{5}{2}x - \frac{11}{2}$ ,  $1 \leq x \leq 5$ .

52. The slope of the line segment joining the points  $(-5, 10)$  and  $(7, -10)$  is  $\frac{-10 - 10}{7 - (-5)} = -\frac{5}{3}$ , so an equation is  $y - 10 = -\frac{5}{3}[x - (-5)]$ . The function is  $f(x) = -\frac{5}{3}x + \frac{5}{3}$ ,  $-5 \leq x \leq 7$ .

53. We need to solve the given equation for  $y$ .  $x + (y - 1)^2 = 0 \Leftrightarrow (y - 1)^2 = -x \Leftrightarrow y - 1 = \pm\sqrt{-x} \Leftrightarrow y = 1 \pm \sqrt{-x}$ . The expression with the positive radical represents the top half of the parabola, and the one with the negative radical represents the bottom half. Hence, we want  $f(x) = 1 - \sqrt{-x}$ . Note that the domain is  $x \leq 0$ .

54.  $x^2 + (y - 2)^2 = 4 \Leftrightarrow (y - 2)^2 = 4 - x^2 \Leftrightarrow y - 2 = \pm\sqrt{4 - x^2} \Leftrightarrow y = 2 \pm \sqrt{4 - x^2}$ . The top half is given by the function  $f(x) = 2 + \sqrt{4 - x^2}$ ,  $-2 \leq x \leq 2$ .

55. For  $0 \leq x \leq 3$ , the graph is the line with slope  $-1$  and  $y$ -intercept  $3$ , that is,  $y = -x + 3$ . For  $3 < x \leq 5$ , the graph is the line with slope  $2$  passing through  $(3, 0)$ ; that is,  $y - 0 = 2(x - 3)$ , or  $y = 2x - 6$ . So the function is

$$f(x) = \begin{cases} -x + 3 & \text{if } 0 \leq x \leq 3 \\ 2x - 6 & \text{if } 3 < x \leq 5 \end{cases}$$

56. For  $-4 \leq x \leq -2$ , the graph is the line with slope  $-\frac{3}{2}$  passing through  $(-2, 0)$ ; that is,  $y - 0 = -\frac{3}{2}[x - (-2)]$ , or  $y = -\frac{3}{2}x - 3$ . For  $-2 < x < 2$ , the graph is the top half of the circle with center  $(0, 0)$  and radius  $2$ . An equation of the circle



is  $x^2 + y^2 = 4$ , so an equation of the top half is  $y = \sqrt{4 - x^2}$ . For  $2 \leq x \leq 4$ , the graph is the line with slope  $\frac{3}{2}$  passing through  $(2, 0)$ ; that is,  $y - 0 = \frac{3}{2}(x - 2)$ , or  $y = \frac{3}{2}x - 3$ . So the function is

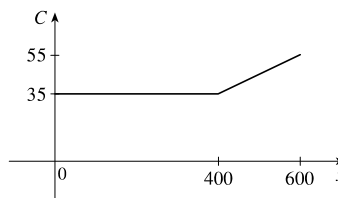
$$f(x) = \begin{cases} -\frac{3}{2}x - 3 & \text{if } -4 \leq x \leq -2 \\ \sqrt{4 - x^2} & \text{if } -2 < x < 2 \\ \frac{3}{2}x - 3 & \text{if } 2 \leq x \leq 4 \end{cases}$$

- 57.** Let the length and width of the rectangle be  $L$  and  $W$ . Then the perimeter is  $2L + 2W = 20$  and the area is  $A = LW$ . Solving the first equation for  $W$  in terms of  $L$  gives  $W = \frac{20 - 2L}{2} = 10 - L$ . Thus,  $A(L) = L(10 - L) = 10L - L^2$ . Since lengths are positive, the domain of  $A$  is  $0 < L < 10$ . If we further restrict  $L$  to be larger than  $W$ , then  $5 < L < 10$  would be the domain.
- 58.** Let the length and width of the rectangle be  $L$  and  $W$ . Then the area is  $LW = 16$ , so that  $W = 16/L$ . The perimeter is  $P = 2L + 2W$ , so  $P(L) = 2L + 2(16/L) = 2L + 32/L$ , and the domain of  $P$  is  $L > 0$ , since lengths must be positive quantities. If we further restrict  $L$  to be larger than  $W$ , then  $L > 4$  would be the domain.
- 59.** Let the length of a side of the equilateral triangle be  $x$ . Then by the Pythagorean Theorem, the height  $y$  of the triangle satisfies  $y^2 + (\frac{1}{2}x)^2 = x^2$ , so that  $y^2 = x^2 - \frac{1}{4}x^2 = \frac{3}{4}x^2$  and  $y = \frac{\sqrt{3}}{2}x$ . Using the formula for the area  $A$  of a triangle,  $A = \frac{1}{2}(\text{base})(\text{height})$ , we obtain  $A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$ , with domain  $x > 0$ .
- 60.** Let the length, width, and height of the closed rectangular box be denoted by  $L$ ,  $W$ , and  $H$ , respectively. The length is twice the width, so  $L = 2W$ . The volume  $V$  of the box is given by  $V = LWH$ . Since  $V = 8$ , we have  $8 = (2W)WH \Rightarrow 8 = 2W^2H \Rightarrow H = \frac{8}{2W^2} = \frac{4}{W^2}$ , and so  $H = f(W) = \frac{4}{W^2}$ .
- 61.** Let each side of the base of the box have length  $x$ , and let the height of the box be  $h$ . Since the volume is 2, we know that  $2 = hx^2$ , so that  $h = 2/x^2$ , and the surface area is  $S = x^2 + 4xh$ . Thus,  $S(x) = x^2 + 4x(2/x^2) = x^2 + (8/x)$ , with domain  $x > 0$ .
- 62.** The area of the window is  $A = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = xh + \frac{\pi x^2}{8}$ , where  $h$  is the height of the rectangular portion of the window. The perimeter is  $P = 2h + x + \frac{1}{2}\pi x = 30 \Leftrightarrow 2h = 30 - x - \frac{1}{2}\pi x \Leftrightarrow h = \frac{1}{4}(60 - 2x - \pi x)$ . Thus,  $A(x) = x \frac{60 - 2x - \pi x}{4} + \frac{\pi x^2}{8} = 15x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2 = 15x - \frac{4}{8}x^2 - \frac{\pi}{8}x^2 = 15x - x^2\left(\frac{\pi + 4}{8}\right)$ . Since the lengths  $x$  and  $h$  must be positive quantities, we have  $x > 0$  and  $h > 0$ . For  $h > 0$ , we have  $2h > 0 \Leftrightarrow 30 - x - \frac{1}{2}\pi x > 0 \Leftrightarrow 60 > 2x + \pi x \Leftrightarrow x < \frac{60}{2 + \pi}$ . Hence, the domain of  $A$  is  $0 < x < \frac{60}{2 + \pi}$ .
- 63.** The height of the box is  $x$  and the length and width are  $L = 20 - 2x$ ,  $W = 12 - 2x$ . Then  $V = LWx$  and so  $V(x) = (20 - 2x)(12 - 2x)(x) = 4(10 - x)(6 - x)(x) = 4x(60 - 16x + x^2) = 4x^3 - 64x^2 + 240x$ . The sides  $L$ ,  $W$ , and  $x$  must be positive. Thus,  $L > 0 \Leftrightarrow 20 - 2x > 0 \Leftrightarrow x < 10$ ;  $W > 0 \Leftrightarrow 12 - 2x > 0 \Leftrightarrow x < 6$ ; and  $x > 0$ . Combining these restrictions gives us the domain  $0 < x < 6$ .

# NOT FOR SALE

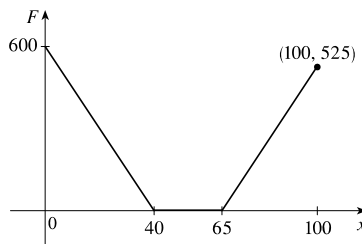
64. We can summarize the monthly cost with a piecewise defined function.

$$C(x) = \begin{cases} 35 & \text{if } 0 \leq x \leq 400 \\ 35 + 0.10(x - 400) & \text{if } x > 400 \end{cases}$$



65. We can summarize the amount of the fine with a piecewise defined function.

$$F(x) = \begin{cases} 15(40 - x) & \text{if } 0 \leq x < 40 \\ 0 & \text{if } 40 \leq x \leq 65 \\ 15(x - 65) & \text{if } x > 65 \end{cases}$$



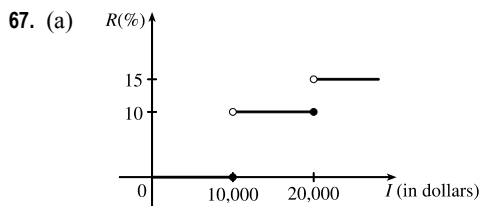
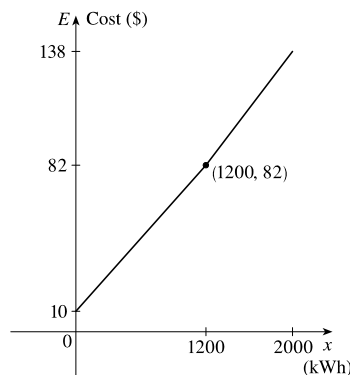
66. For the first 1200 kWh,  $E(x) = 10 + 0.06x$ .

For usage over 1200 kWh, the cost is

$$E(x) = 10 + 0.06(1200) + 0.07(x - 1200) = 82 + 0.07(x - 1200).$$

Thus,

$$E(x) = \begin{cases} 10 + 0.06x & \text{if } 0 \leq x \leq 1200 \\ 82 + 0.07(x - 1200) & \text{if } x > 1200 \end{cases}$$



(b) On \$14,000, tax is assessed on \$4000, and  $10\%(\$4000) = \$400$ .

On \$26,000, tax is assessed on \$16,000, and

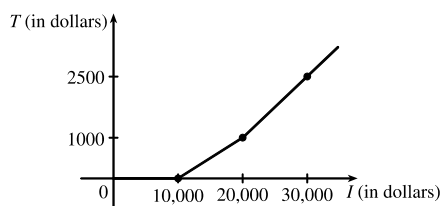
$$10\%(\$10,000) + 15\%(\$6000) = \$1000 + \$900 = \$1900.$$

(c) As in part (b), there is \$1000 tax assessed on \$20,000 of income, so the graph of  $T$  is a line segment from  $(10,000, 0)$  to  $(20,000, 1000)$ .

The tax on \$30,000 is \$2500, so the graph of  $T$  for  $x > 20,000$  is

the ray with initial point  $(20,000, 1000)$  that passes through

$(30,000, 2500)$ .



68. One example is the amount paid for cable or telephone system repair in the home, usually measured to the nearest quarter hour. Another example is the amount paid by a student in tuition fees, if the fees vary according to the number of credits for which the student has registered.
69.  $f$  is an odd function because its graph is symmetric about the origin.  $g$  is an even function because its graph is symmetric with respect to the  $y$ -axis.

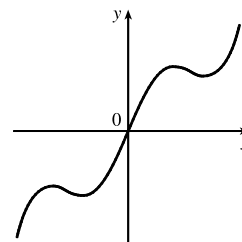
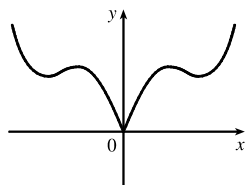
70.  $f$  is not an even function since it is not symmetric with respect to the  $y$ -axis.  $f$  is not an odd function since it is not symmetric about the origin. Hence,  $f$  is *neither* even nor odd.  $g$  is an even function because its graph is symmetric with respect to the  $y$ -axis.

71. (a) Because an even function is symmetric with respect to the  $y$ -axis, and the point  $(5, 3)$  is on the graph of this even function, the point  $(-5, 3)$  must also be on its graph.

(b) Because an odd function is symmetric with respect to the origin, and the point  $(5, 3)$  is on the graph of this odd function, the point  $(-5, -3)$  must also be on its graph.

72. (a) If  $f$  is even, we get the rest of the graph by reflecting about the  $y$ -axis.

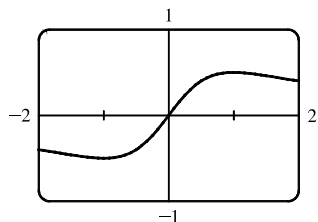
(b) If  $f$  is odd, we get the rest of the graph by rotating  $180^\circ$  about the origin.



73.  $f(x) = \frac{x}{x^2 + 1}$ .

$$f(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} = -\frac{x}{x^2 + 1} = -f(x).$$

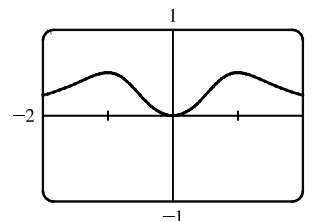
Since  $f(-x) = -f(x)$ ,  $f$  is an odd function.



74.  $f(x) = \frac{x^2}{x^4 + 1}$ .

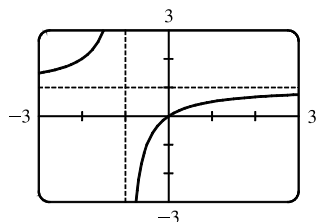
$$f(-x) = \frac{(-x)^2}{(-x)^4 + 1} = \frac{x^2}{x^4 + 1} = f(x).$$

Since  $f(-x) = f(x)$ ,  $f$  is an even function.



75.  $f(x) = \frac{x}{x+1}$ , so  $f(-x) = \frac{-x}{-x+1} = \frac{x}{x-1}$ .

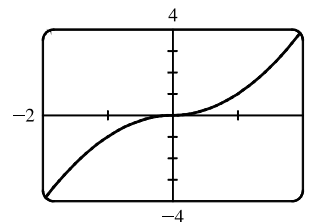
Since this is neither  $f(x)$  nor  $-f(x)$ , the function  $f$  is neither even nor odd.



76.  $f(x) = x|x|$ .

$$\begin{aligned} f(-x) &= (-x)|-x| = (-x)|x| = -(x|x|) \\ &= -f(x) \end{aligned}$$

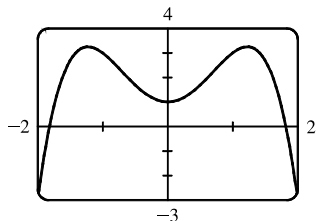
Since  $f(-x) = -f(x)$ ,  $f$  is an odd function.



77.  $f(x) = 1 + 3x^2 - x^4$ .

$$f(-x) = 1 + 3(-x)^2 - (-x)^4 = 1 + 3x^2 - x^4 = f(x).$$

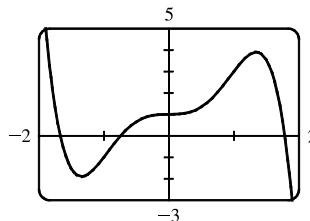
Since  $f(-x) = f(x)$ ,  $f$  is an even function.



78.  $f(x) = 1 + 3x^3 - x^5$ , so

$$\begin{aligned} f(-x) &= 1 + 3(-x)^3 - (-x)^5 = 1 + 3(-x^3) - (-x^5) \\ &= 1 - 3x^3 + x^5 \end{aligned}$$

Since this is neither  $f(x)$  nor  $-f(x)$ , the function  $f$  is neither even nor odd.



79. (i) If  $f$  and  $g$  are both even functions, then  $f(-x) = f(x)$  and  $g(-x) = g(x)$ . Now

$$(f + g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f + g)(x), \text{ so } f + g \text{ is an even function.}$$

(ii) If  $f$  and  $g$  are both odd functions, then  $f(-x) = -f(x)$  and  $g(-x) = -g(x)$ . Now

$$(f + g)(-x) = f(-x) + g(-x) = -f(x) + [-g(x)] = -[f(x) + g(x)] = -(f + g)(x), \text{ so } f + g \text{ is an odd function.}$$

(iii) If  $f$  is an even function and  $g$  is an odd function, then  $(f + g)(-x) = f(-x) + g(-x) = f(x) + [-g(x)] = f(x) - g(x)$ , which is not  $(f + g)(x)$  nor  $-(f + g)(x)$ , so  $f + g$  is neither even nor odd. (Exception: if  $f$  is the zero function, then  $f + g$  will be odd. If  $g$  is the zero function, then  $f + g$  will be even.)

80. (i) If  $f$  and  $g$  are both even functions, then  $f(-x) = f(x)$  and  $g(-x) = g(x)$ . Now

$$(fg)(-x) = f(-x)g(-x) = f(x)g(x) = (fg)(x), \text{ so } fg \text{ is an even function.}$$

(ii) If  $f$  and  $g$  are both odd functions, then  $f(-x) = -f(x)$  and  $g(-x) = -g(x)$ . Now

$$(fg)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (fg)(x), \text{ so } fg \text{ is an even function.}$$

(iii) If  $f$  is an even function and  $g$  is an odd function, then

$$(fg)(-x) = f(-x)g(-x) = f(x)[-g(x)] = -[f(x)g(x)] = -(fg)(x), \text{ so } fg \text{ is an odd function.}$$

## 1.2 Mathematical Models: A Catalog of Essential Functions

1. (a)  $f(x) = \log_2 x$  is a logarithmic function.

(b)  $g(x) = \sqrt[4]{x}$  is a root function with  $n = 4$ .

(c)  $h(x) = \frac{2x^3}{1 - x^2}$  is a rational function because it is a ratio of polynomials.

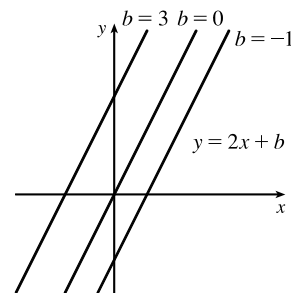
(d)  $u(t) = 1 - 1.1t + 2.54t^2$  is a polynomial of degree 2 (also called a *quadratic function*).

(e)  $v(t) = 5^t$  is an exponential function.

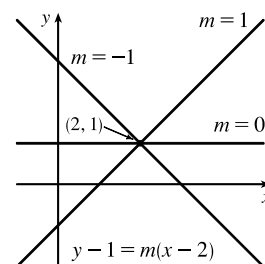
(f)  $w(\theta) = \sin \theta \cos^2 \theta$  is a trigonometric function.

2. (a)  $y = \pi^x$  is an exponential function (notice that  $x$  is the *exponent*).  
 (b)  $y = x^\pi$  is a power function (notice that  $x$  is the *base*).  
 (c)  $y = x^2(2 - x^3) = 2x^2 - x^5$  is a polynomial of degree 5.  
 (d)  $y = \tan t - \cos t$  is a trigonometric function.  
 (e)  $y = s/(1 + s)$  is a rational function because it is a ratio of polynomials.  
 (f)  $y = \sqrt{x^3 - 1}/(1 + \sqrt[3]{x})$  is an algebraic function because it involves polynomials and roots of polynomials.
3. We notice from the figure that  $g$  and  $h$  are even functions (symmetric with respect to the  $y$ -axis) and that  $f$  is an odd function (symmetric with respect to the origin). So (b)  $[y = x^5]$  must be  $f$ . Since  $g$  is flatter than  $h$  near the origin, we must have (c)  $[y = x^8]$  matched with  $g$  and (a)  $[y = x^2]$  matched with  $h$ .
4. (a) The graph of  $y = 3x$  is a line (choice  $G$ ).  
 (b)  $y = 3^x$  is an exponential function (choice  $f$ ).  
 (c)  $y = x^3$  is an odd polynomial function or power function (choice  $F$ ).  
 (d)  $y = \sqrt[3]{x} = x^{1/3}$  is a root function (choice  $g$ ).
5. The denominator cannot equal 0, so  $1 - \sin x \neq 0 \Leftrightarrow \sin x \neq 1 \Leftrightarrow x \neq \frac{\pi}{2} + 2n\pi$ . Thus, the domain of  $f(x) = \frac{\cos x}{1 - \sin x}$  is  $\{x \mid x \neq \frac{\pi}{2} + 2n\pi, n \text{ an integer}\}$ .
6. The denominator cannot equal 0, so  $1 - \tan x \neq 0 \Leftrightarrow \tan x \neq 1 \Leftrightarrow x \neq \frac{\pi}{4} + n\pi$ . The tangent function is not defined if  $x \neq \frac{\pi}{2} + n\pi$ . Thus, the domain of  $g(x) = \frac{1}{1 - \tan x}$  is  $\{x \mid x \neq \frac{\pi}{4} + n\pi, x \neq \frac{\pi}{2} + n\pi, n \text{ an integer}\}$ .

7. (a) An equation for the family of linear functions with slope 2 is  $y = f(x) = 2x + b$ , where  $b$  is the  $y$ -intercept.

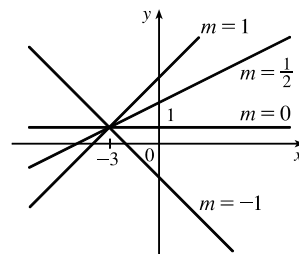


- (b)  $f(2) = 1$  means that the point  $(2, 1)$  is on the graph of  $f$ . We can use the point-slope form of a line to obtain an equation for the family of linear functions through the point  $(2, 1)$ .  $y - 1 = m(x - 2)$ , which is equivalent to  $y = mx + (1 - 2m)$  in slope-intercept form.

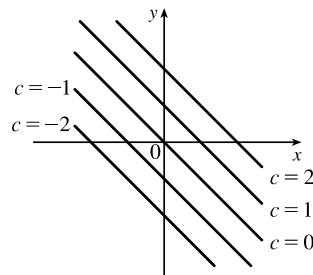


- (c) To belong to both families, an equation must have slope  $m = 2$ , so the equation in part (b),  $y = mx + (1 - 2m)$ , becomes  $y = 2x - 3$ . It is the *only* function that belongs to both families.

8. All members of the family of linear functions  $f(x) = 1 + m(x + 3)$  have graphs that are lines passing through the point  $(-3, 1)$ .



9. All members of the family of linear functions  $f(x) = c - x$  have graphs that are lines with slope  $-1$ . The  $y$ -intercept is  $c$ .



10. The vertex of the parabola on the left is  $(3, 0)$ , so an equation is  $y = a(x - 3)^2 + 0$ . Since the point  $(4, 2)$  is on the parabola, we'll substitute 4 for  $x$  and 2 for  $y$  to find  $a$ .  $2 = a(4 - 3)^2 \Rightarrow a = 2$ , so an equation is  $f(x) = 2(x - 3)^2$ .

The  $y$ -intercept of the parabola on the right is  $(0, 1)$ , so an equation is  $y = ax^2 + bx + 1$ . Since the points  $(-2, 2)$  and  $(1, -2.5)$  are on the parabola, we'll substitute  $-2$  for  $x$  and 2 for  $y$  as well as 1 for  $x$  and  $-2.5$  for  $y$  to obtain two equations with the unknowns  $a$  and  $b$ .

$$(-2, 2): \quad 2 = 4a - 2b + 1 \Rightarrow 4a - 2b = 1 \quad (1)$$

$$(1, -2.5): \quad -2.5 = a + b + 1 \Rightarrow a + b = -3.5 \quad (2)$$

$2 \cdot (2) + (1)$  gives us  $6a = -6 \Rightarrow a = -1$ . From  $(2)$ ,  $-1 + b = -3.5 \Rightarrow b = -2.5$ , so an equation is  $g(x) = -x^2 - 2.5x + 1$ .

11. Since  $f(-1) = f(0) = f(2) = 0$ ,  $f$  has zeros of  $-1, 0$ , and  $2$ , so an equation for  $f$  is  $f(x) = a[x - (-1)](x - 0)(x - 2)$ , or  $f(x) = ax(x + 1)(x - 2)$ . Because  $f(1) = 6$ , we'll substitute 1 for  $x$  and 6 for  $f(x)$ .

$$6 = a(1)(2)(-1) \Rightarrow -2a = 6 \Rightarrow a = -3, \text{ so an equation for } f \text{ is } f(x) = -3x(x + 1)(x - 2).$$

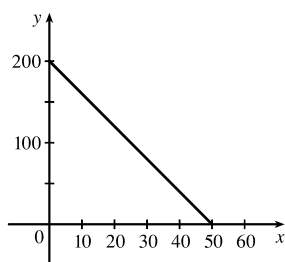
12. (a) For  $T = 0.02t + 8.50$ , the slope is 0.02, which means that the average surface temperature of the world is increasing at a rate of  $0.02^\circ\text{C}$  per year. The  $T$ -intercept is 8.50, which represents the average surface temperature in  $^\circ\text{C}$  in the year 1900.

(b)  $t = 2100 - 1900 = 200 \Rightarrow T = 0.02(200) + 8.50 = 12.50^\circ\text{C}$

13. (a)  $D = 200$ , so  $c = 0.0417D(a + 1) = 0.0417(200)(a + 1) = 8.34a + 8.34$ . The slope is 8.34, which represents the change in mg of the dosage for a child for each change of 1 year in age.

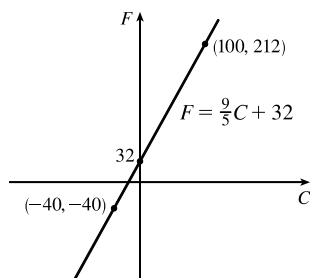
(b) For a newborn,  $a = 0$ , so  $c = 8.34$  mg.

14. (a)



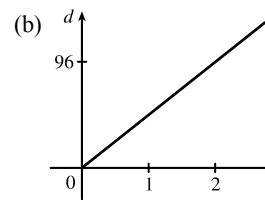
(b) The slope of  $-4$  means that for each increase of 1 dollar for a rental space, the number of spaces rented *decreases* by 4. The  $y$ -intercept of 200 is the number of spaces that would be occupied if there were no charge for each space. The  $x$ -intercept of 50 is the smallest rental fee that results in no spaces rented.

15. (a)



(b) The slope of  $\frac{9}{5}$  means that  $F$  increases  $\frac{9}{5}$  degrees for each increase of  $1^\circ\text{C}$ . (Equivalently,  $F$  increases by 9 when  $C$  increases by 5 and  $F$  decreases by 9 when  $C$  decreases by 5.) The  $F$ -intercept of 32 is the Fahrenheit temperature corresponding to a Celsius temperature of 0.

16. (a) Let  $d$  = distance traveled (in miles) and  $t$  = time elapsed (in hours). At  $t = 0$ ,  $d = 0$  and at  $t = 50 \text{ minutes} = 50 \cdot \frac{1}{60} = \frac{5}{6} \text{ h}$ ,  $d = 40$ . Thus we have two points:  $(0, 0)$  and  $(\frac{5}{6}, 40)$ , so  $m = \frac{40 - 0}{\frac{5}{6} - 0} = 48$  and so  $d = 48t$ .



(c) The slope is 48 and represents the car's speed in mi/h.

17. (a) Using  $N$  in place of  $x$  and  $T$  in place of  $y$ , we find the slope to be  $\frac{T_2 - T_1}{N_2 - N_1} = \frac{80 - 70}{173 - 113} = \frac{10}{60} = \frac{1}{6}$ . So a linear equation is  $T - 80 = \frac{1}{6}(N - 173) \Leftrightarrow T - 80 = \frac{1}{6}N - \frac{173}{6} \Leftrightarrow T = \frac{1}{6}N + \frac{307}{6}$  [ $\frac{307}{6} = 51.1\bar{6}$ ].

(b) The slope of  $\frac{1}{6}$  means that the temperature in Fahrenheit degrees increases one-sixth as rapidly as the number of cricket chirps per minute. Said differently, each increase of 6 cricket chirps per minute corresponds to an increase of  $1^\circ\text{F}$ .

(c) When  $N = 150$ , the temperature is given approximately by  $T = \frac{1}{6}(150) + \frac{307}{6} = 76.1\bar{6}^\circ\text{F} \approx 76^\circ\text{F}$ .

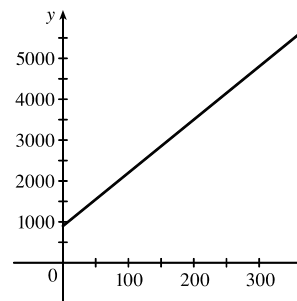
18. (a) Let  $x$  denote the number of chairs produced in one day and  $y$  the associated cost. Using the points  $(100, 2200)$  and  $(300, 4800)$ , we get the slope

$$\frac{4800 - 2200}{300 - 100} = \frac{2600}{200} = 13. \text{ So } y - 2200 = 13(x - 100) \Leftrightarrow$$

$$y = 13x + 900.$$

(b) The slope of the line in part (a) is 13 and it represents the cost (in dollars) of producing each additional chair.

(c) The  $y$ -intercept is 900 and it represents the fixed daily costs of operating the factory.



19. (a) We are given  $\frac{\text{change in pressure}}{10 \text{ feet change in depth}} = \frac{4.34}{10} = 0.434$ . Using  $P$  for pressure and  $d$  for depth with the point  $(d, P) = (0, 15)$ , we have the slope-intercept form of the line,  $P = 0.434d + 15$ .

# NOT FOR SALE

24 □ CHAPTER 1 FUNCTIONS AND MODELS

(b) When  $P = 100$ , then  $100 = 0.434d + 15 \Leftrightarrow 0.434d = 85 \Leftrightarrow d = \frac{85}{0.434} \approx 195.85$  feet. Thus, the pressure is 100 lb/in<sup>2</sup> at a depth of approximately 196 feet.

20. (a) Using  $d$  in place of  $x$  and  $C$  in place of  $y$ , we find the slope to be  $\frac{C_2 - C_1}{d_2 - d_1} = \frac{460 - 380}{800 - 480} = \frac{80}{320} = \frac{1}{4}$ .

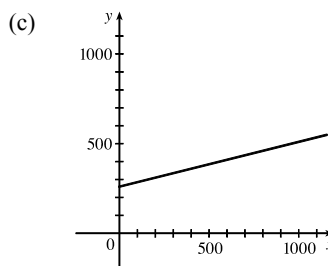
So a linear equation is  $C - 460 = \frac{1}{4}(d - 800) \Leftrightarrow C - 460 = \frac{1}{4}d - 200 \Leftrightarrow C = \frac{1}{4}d + 260$ .

(b) Letting  $d = 1500$  we get  $C = \frac{1}{4}(1500) + 260 = 635$ .

The cost of driving 1500 miles is \$635.

(d) The  $y$ -intercept represents the fixed cost, \$260.

(e) A linear function gives a suitable model in this situation because you have fixed monthly costs such as insurance and car payments, as well as costs that increase as you drive, such as gasoline, oil, and tires, and the cost of these for each additional mile driven is a constant.



The slope of the line represents the cost per mile, \$0.25.

21. (a) The data appear to be periodic and a sine or cosine function would make the best model. A model of the form

$$f(x) = a \cos(bx) + c \text{ seems appropriate.}$$

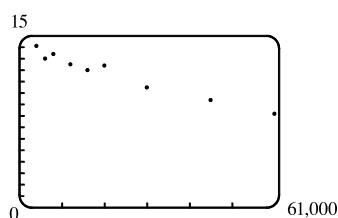
(b) The data appear to be decreasing in a linear fashion. A model of the form  $f(x) = mx + b$  seems appropriate.

22. (a) The data appear to be increasing exponentially. A model of the form  $f(x) = a \cdot b^x$  or  $f(x) = a \cdot b^x + c$  seems appropriate.

(b) The data appear to be decreasing similarly to the values of the reciprocal function. A model of the form  $f(x) = a/x$  seems appropriate.

Exercises 23–28: Some values are given to many decimal places. These are the results given by several computer algebra systems — rounding is left to the reader.

23. (a)

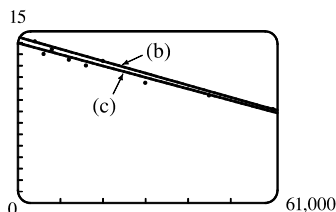


A linear model does seem appropriate.

(b) Using the points (4000, 14.1) and (60,000, 8.2), we obtain

$$y - 14.1 = \frac{8.2 - 14.1}{60,000 - 4000}(x - 4000) \text{ or, equivalently,}$$

$$y \approx -0.000105357x + 14.521429.$$



(c) Using a computing device, we obtain the least squares regression line  $y = -0.0000997855x + 13.950764$ .

The following commands and screens illustrate how to find the least squares regression line on a TI-84 Plus.



# NOT FOR SALE

Enter the data into list one (L1) and list two (L2). Press **STAT** **1** to enter the editor.

L1	L2	L3	1
4000	14.1		
6000	13		
8000	13.4		
12000	12.5		
16000	12.5		
20000	12.4		
30000	10.5		

L1	L2	L3	2
12000	12.5		
16000	12		
20000	12.4		
30000	10.5		
45000	8.4		
60000	8.2		

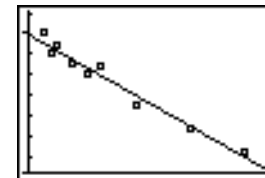
Find the regression line and store it in  $Y_1$ . Press **2nd** **QUIT** **STAT** **►** **4** **VARS** **►** **1** **1** **ENTER**.

LinReg(ax+b) $Y_1$	LinReg y=ax+b a=-9.978546E-5 b=13.95076408	Plot1 Plot2 Plot3 $\checkmark$ Y1 -9.978545618 7893E-5X+13.9507 64077085 Y2= Y3= Y4= Y5=
--------------------	---	---

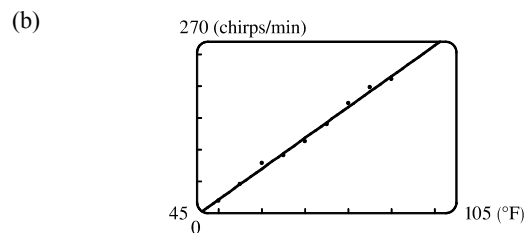
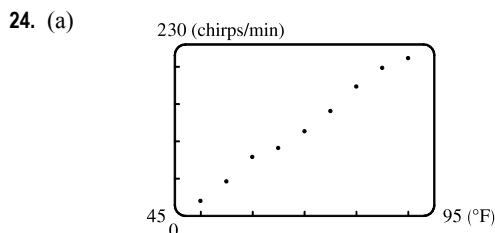
Note from the last figure that the regression line has been stored in  $Y_1$  and that Plot1 has been turned on (Plot1 is highlighted). You can turn on Plot1 from the Y= menu by placing the cursor on Plot1 and pressing **ENTER** or by pressing **2nd** **STAT PLOT** **1** **ENTER**.

2nd STAT PLOTS 1:Plot1...On L1 L2 2:Plot2...Off L1 L2 3:Plot3...Off L1 L2 4:PlotsOff	Plot1 Plot2 Plot3 On Off Type: <input checked="" type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> Xlist:L1 Ylist:L2 Mark: <input checked="" type="checkbox"/> +
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Now press **ZOOM** **9** to produce a graph of the data and the regression line. Note that choice 9 of the ZOOM menu automatically selects a window that displays all of the data.

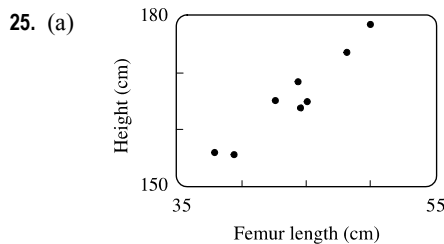


- (d) When  $x = 25,000$ ,  $y \approx 11.456$ ; or about 11.5 per 100 population.
- (e) When  $x = 80,000$ ,  $y \approx 5.968$ ; or about a 6% chance.
- (f) When  $x = 200,000$ ,  $y$  is negative, so the model does not apply.

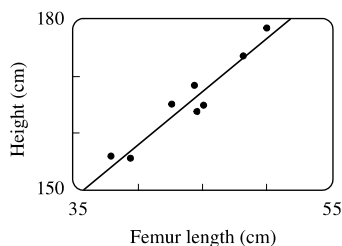


Using a computing device, we obtain the least squares regression line  $y = 4.856x - 220.96$ .

- (c) When  $x = 100^\circ\text{F}$ ,  $y = 264.7 \approx 265$  chirps/min.



(b) Using a computing device, we obtain the regression line  
 $y = 1.88074x + 82.64974$ .

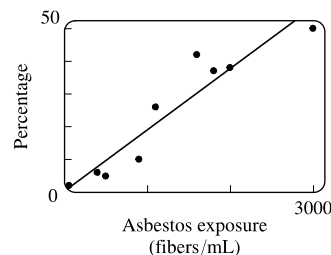


(c) When  $x = 53$  cm,  $y \approx 182.3$  cm.

26. (a) Using a computing device, we obtain the regression line  $y = 0.01879x + 0.30480$ .

(b) The regression line appears to be a suitable model for the data.

(c) The  $y$ -intercept represents the percentage of laboratory rats that develop lung tumors when *not* exposed to asbestos fibers.



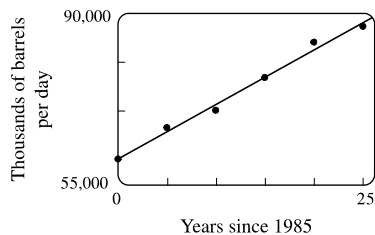
27. (a) See the scatter plot in part (b). A linear model seems appropriate.

(b) Using a computing device, we obtain the regression line

$$y = 1116.64x + 60,188.33.$$

(c) For 2002,  $x = 17$  and  $y \approx 79,171$  thousands of barrels per day.

For 2012,  $x = 27$  and  $y \approx 90,338$  thousands of barrels per day.



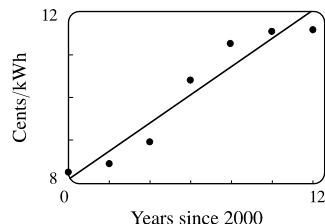
28. (a) See the scatter plot in part (b). A linear model seems appropriate.

(b) Using a computing device, we obtain the regression line

$$y = 0.33089x + 8.07321.$$

(c) For 2005,  $x = 5$  and  $y \approx 9.73$  cents/kWh. For 2013,  $x = 13$  and

$y \approx 12.37$  cents/kWh.



29. If  $x$  is the original distance from the source, then the illumination is  $f(x) = kx^{-2} = k/x^2$ . Moving halfway to the lamp gives us an illumination of  $f(\frac{1}{2}x) = k(\frac{1}{2}x)^{-2} = k(2/x)^2 = 4(k/x^2)$ , so the light is 4 times as bright.

30. (a) If  $A = 60$ , then  $S = 0.7A^{0.3} \approx 2.39$ , so you would expect to find 2 species of bats in that cave.

(b)  $S = 4 \Rightarrow 4 = 0.7A^{0.3} \Rightarrow \frac{40}{7} = A^{3/10} \Rightarrow A = (\frac{40}{7})^{10/3} \approx 333.6$ , so we estimate the surface area of the cave to be  $334 \text{ m}^2$ .

31. (a) Using a computing device, we obtain a power function  $N = cA^b$ , where  $c \approx 3.1046$  and  $b \approx 0.308$ .

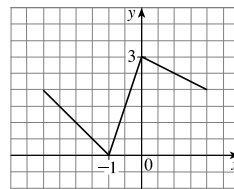
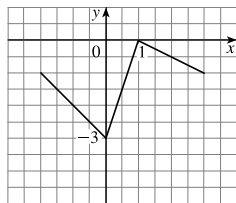
(b) If  $A = 291$ , then  $N = cA^b \approx 17.8$ , so you would expect to find 18 species of reptiles and amphibians on Dominica.

32. (a)  $T = 1.000431227d^{1.499528750}$

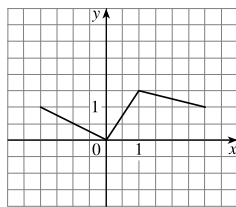
- (b) The power model in part (a) is approximately  $T = d^{1.5}$ . Squaring both sides gives us  $T^2 = d^3$ , so the model matches Kepler's Third Law,  $T^2 = kd^3$ .

## 1.3 New Functions from Old Functions

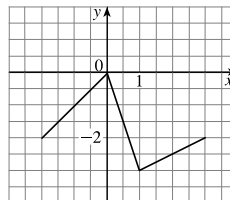
1. (a) If the graph of  $f$  is shifted 3 units upward, its equation becomes  $y = f(x) + 3$ .  
 (b) If the graph of  $f$  is shifted 3 units downward, its equation becomes  $y = f(x) - 3$ .  
 (c) If the graph of  $f$  is shifted 3 units to the right, its equation becomes  $y = f(x - 3)$ .  
 (d) If the graph of  $f$  is shifted 3 units to the left, its equation becomes  $y = f(x + 3)$ .  
 (e) If the graph of  $f$  is reflected about the  $x$ -axis, its equation becomes  $y = -f(x)$ .  
 (f) If the graph of  $f$  is reflected about the  $y$ -axis, its equation becomes  $y = f(-x)$ .  
 (g) If the graph of  $f$  is stretched vertically by a factor of 3, its equation becomes  $y = 3f(x)$ .  
 (h) If the graph of  $f$  is shrunk vertically by a factor of 3, its equation becomes  $y = \frac{1}{3}f(x)$ .
2. (a) To obtain the graph of  $y = f(x) + 8$  from the graph of  $y = f(x)$ , shift the graph 8 units upward.  
 (b) To obtain the graph of  $y = f(x + 8)$  from the graph of  $y = f(x)$ , shift the graph 8 units to the left.  
 (c) To obtain the graph of  $y = 8f(x)$  from the graph of  $y = f(x)$ , stretch the graph vertically by a factor of 8.  
 (d) To obtain the graph of  $y = f(8x)$  from the graph of  $y = f(x)$ , shrink the graph horizontally by a factor of 8.  
 (e) To obtain the graph of  $y = -f(x) - 1$  from the graph of  $y = f(x)$ , first reflect the graph about the  $x$ -axis, and then shift it 1 unit downward.  
 (f) To obtain the graph of  $y = 8f(\frac{1}{8}x)$  from the graph of  $y = f(x)$ , stretch the graph horizontally and vertically by a factor of 8.
3. (a) (graph 3) The graph of  $f$  is shifted 4 units to the right and has equation  $y = f(x - 4)$ .  
 (b) (graph 1) The graph of  $f$  is shifted 3 units upward and has equation  $y = f(x) + 3$ .  
 (c) (graph 4) The graph of  $f$  is shrunk vertically by a factor of 3 and has equation  $y = \frac{1}{3}f(x)$ .  
 (d) (graph 5) The graph of  $f$  is shifted 4 units to the left and reflected about the  $x$ -axis. Its equation is  $y = -f(x + 4)$ .  
 (e) (graph 2) The graph of  $f$  is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is  $y = 2f(x + 6)$ .
4. (a)  $y = f(x) - 3$ : Shift the graph of  $f$  3 units down.      (b)  $y = f(x + 1)$ : Shift the graph of  $f$  1 unit to the left.



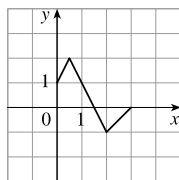
(c)  $y = \frac{1}{2}f(x)$ : Shrink the graph of  $f$  vertically by a factor of 2.



(d)  $y = -f(x)$ : Reflect the graph of  $f$  about the  $x$ -axis.

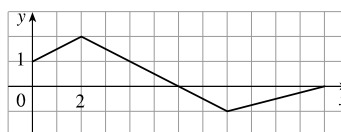


5. (a) To graph  $y = f(2x)$  we shrink the graph of  $f$  horizontally by a factor of 2.



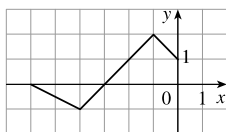
The point  $(4, -1)$  on the graph of  $f$  corresponds to the point  $(\frac{1}{2} \cdot 4, -1) = (2, -1)$ .

(b) To graph  $y = f(\frac{1}{2}x)$  we stretch the graph of  $f$  horizontally by a factor of 2.



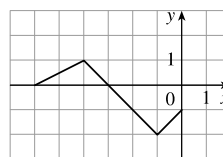
The point  $(4, -1)$  on the graph of  $f$  corresponds to the point  $(2 \cdot 4, -1) = (8, -1)$ .

(c) To graph  $y = f(-x)$  we reflect the graph of  $f$  about the  $y$ -axis.



The point  $(4, -1)$  on the graph of  $f$  corresponds to the point  $(-1 \cdot 4, -1) = (-4, -1)$ .

(d) To graph  $y = -f(-x)$  we reflect the graph of  $f$  about the  $y$ -axis, then about the  $x$ -axis.



The point  $(4, -1)$  on the graph of  $f$  corresponds to the point  $(-1 \cdot 4, -1 \cdot -1) = (-4, 1)$ .

6. The graph of  $y = f(x) = \sqrt{3x - x^2}$  has been shifted 2 units to the right and stretched vertically by a factor of 2.

Thus, a function describing the graph is

$$y = 2f(x - 2) = 2\sqrt{3(x - 2) - (x - 2)^2} = 2\sqrt{3x - 6 - (x^2 - 4x + 4)} = 2\sqrt{-x^2 + 7x - 10}$$

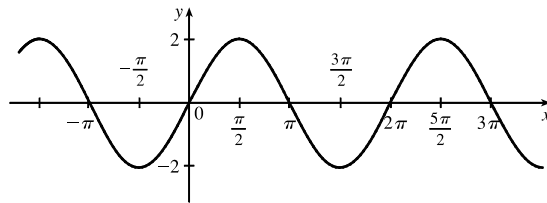
7. The graph of  $y = f(x) = \sqrt{3x - x^2}$  has been shifted 4 units to the left, reflected about the  $x$ -axis, and shifted downward 1 unit. Thus, a function describing the graph is

$$y = \underbrace{-1 \cdot}_{\text{reflect about } x\text{-axis}} \underbrace{f(x + 4)}_{\text{shift 4 units left}} \underbrace{- 1}_{\text{shift 1 unit left}}$$

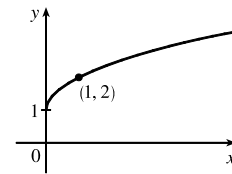
This function can be written as

$$\begin{aligned} y &= -f(x + 4) - 1 = -\sqrt{3(x + 4) - (x + 4)^2} - 1 \\ &= -\sqrt{3x + 12 - (x^2 + 8x + 16)} - 1 = -\sqrt{-x^2 - 5x - 4} - 1 \end{aligned}$$

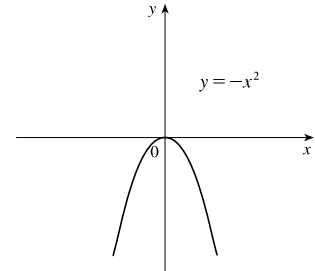
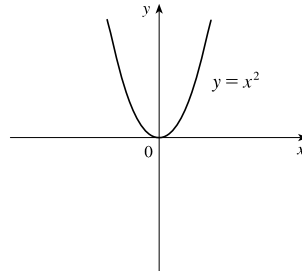
8. (a) The graph of  $y = 2 \sin x$  can be obtained from the graph of  $y = \sin x$  by stretching it vertically by a factor of 2.



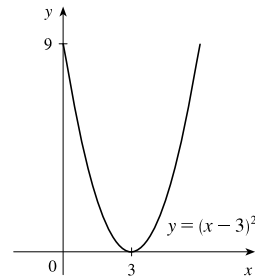
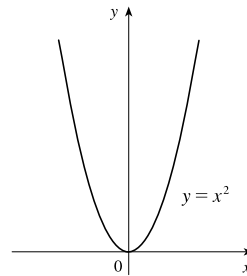
- (b) The graph of  $y = 1 + \sqrt{x}$  can be obtained from the graph of  $y = \sqrt{x}$  by shifting it upward 1 unit.



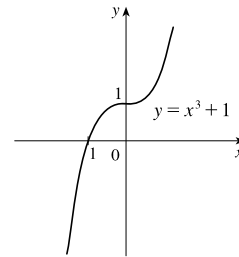
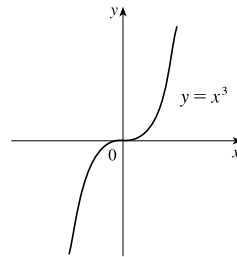
9.  $y = -x^2$ : Start with the graph of  $y = x^2$  and reflect about the  $x$ -axis.



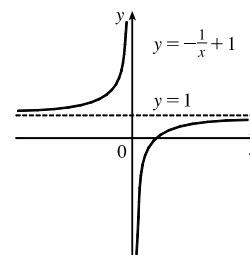
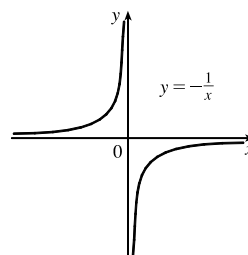
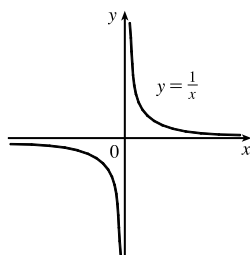
10.  $y = (x - 3)^2$ : Start with the graph of  $y = x^2$  and shift 3 units to the right.



11.  $y = x^3 + 1$ : Start with the graph of  $y = x^3$  and shift upward 1 unit.

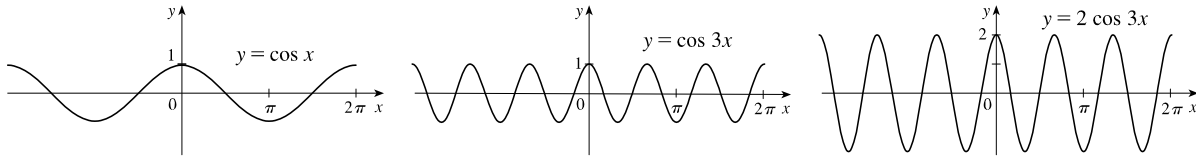


12.  $y = 1 - \frac{1}{x} = -\frac{1}{x} + 1$ : Start with the graph of  $y = \frac{1}{x}$ , reflect about the  $x$ -axis, and shift upward 1 unit.

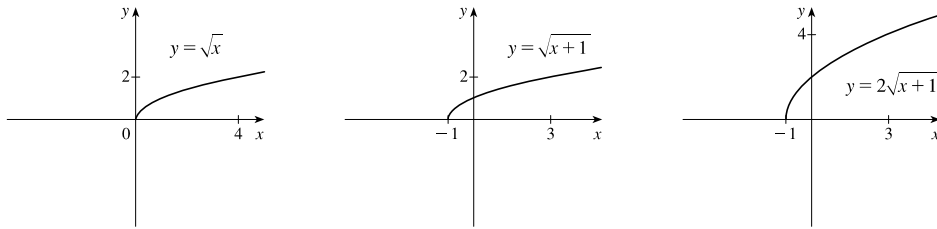


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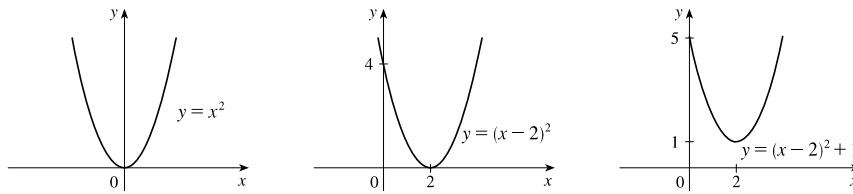
13.  $y = 2 \cos 3x$ : Start with the graph of  $y = \cos x$ , compress horizontally by a factor of 3, and then stretch vertically by a factor of 2.



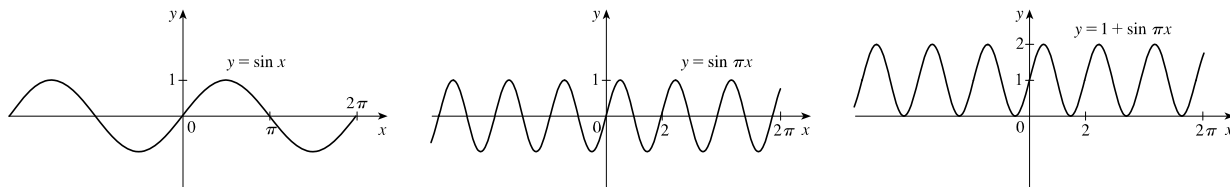
14.  $y = 2\sqrt{x+1}$ : Start with the graph of  $y = \sqrt{x}$ , shift 1 unit to the left, and then stretch vertically by a factor of 2.



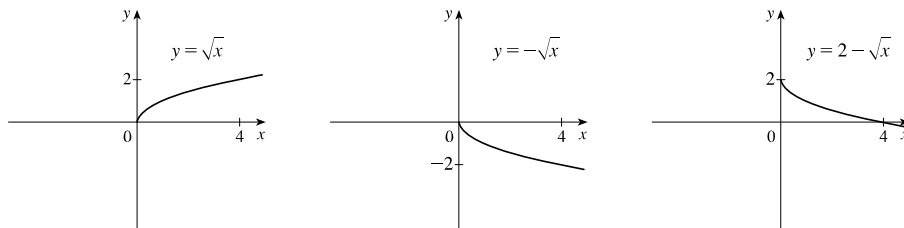
15.  $y = x^2 - 4x + 5 = (x^2 - 4x + 4) + 1 = (x - 2)^2 + 1$ : Start with the graph of  $y = x^2$ , shift 2 units to the right, and then shift upward 1 unit.



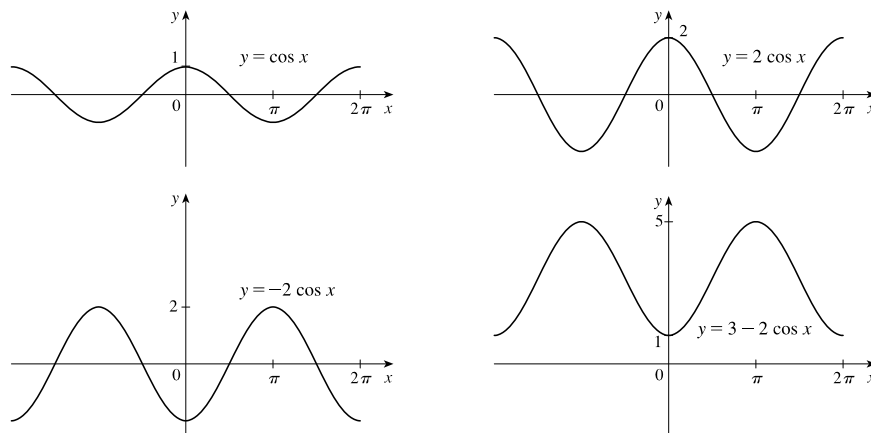
16.  $y = 1 + \sin \pi x$ : Start with the graph of  $y = \sin x$ , compress horizontally by a factor of  $\pi$ , and then shift upward 1 unit.



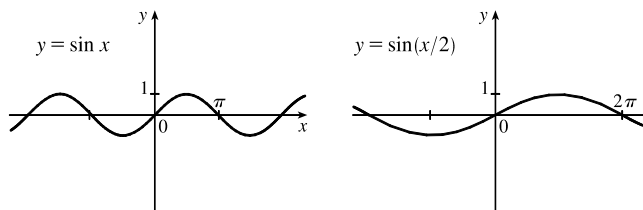
17.  $y = 2 - \sqrt{x}$ : Start with the graph of  $y = \sqrt{x}$ , reflect about the  $x$ -axis, and then shift 2 units upward.



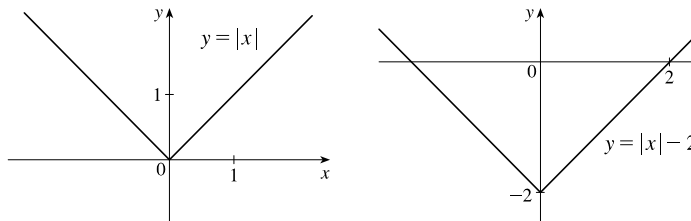
18.  $y = 3 - 2 \cos x$ : Start with the graph of  $y = \cos x$ , stretch vertically by a factor of 2, reflect about the  $x$ -axis, and then shift 3 units upward.



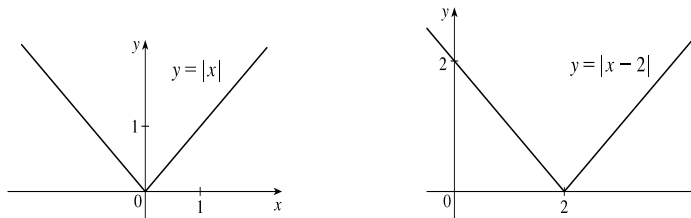
19.  $y = \sin(x/2)$ : Start with the graph of  $y = \sin x$  and stretch horizontally by a factor of 2.



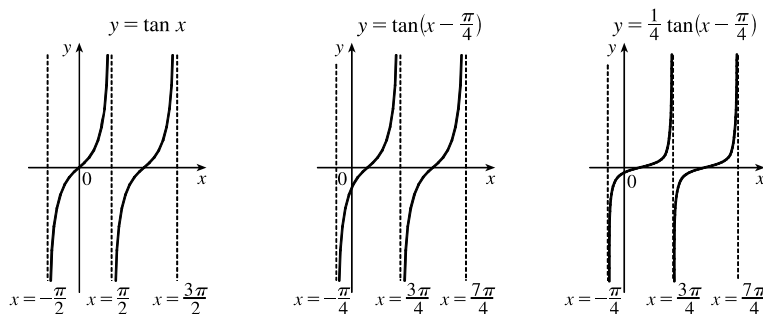
20.  $y = |x| - 2$ : Start with the graph of  $y = |x|$  and shift 2 units downward.



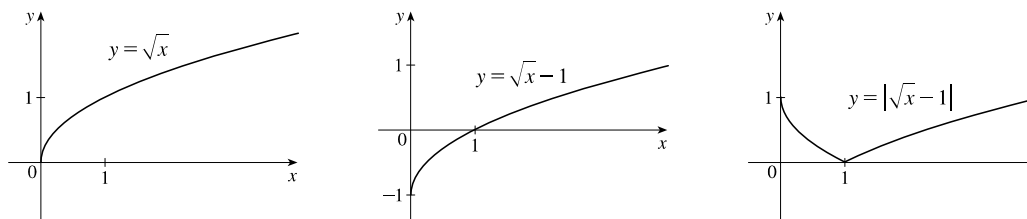
21.  $y = |x - 2|$ : Start with the graph of  $y = |x|$  and shift 2 units to the right.



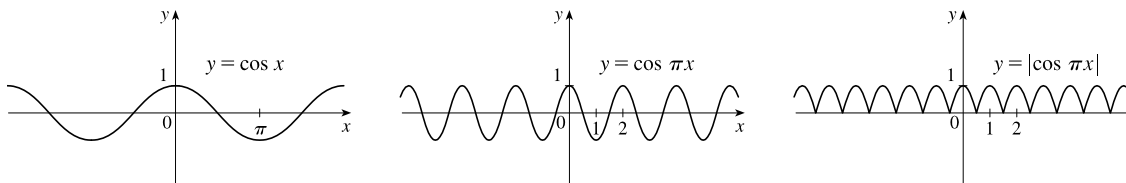
22.  $y = \frac{1}{4} \tan(x - \frac{\pi}{4})$ : Start with the graph of  $y = \tan x$ , shift  $\frac{\pi}{4}$  units to the right, and then compress vertically by a factor of 4.



23.  $y = |\sqrt{x} - 1|$ : Start with the graph of  $y = \sqrt{x}$ , shift it 1 unit downward, and then reflect the portion of the graph below the  $x$ -axis about the  $x$ -axis.



24.  $y = |\cos \pi x|$ : Start with the graph of  $y = \cos x$ , shrink it horizontally by a factor of  $\pi$ , and reflect all the parts of the graph below the  $x$ -axis about the  $x$ -axis.



25. This is just like the solution to Example 4 except the amplitude of the curve (the  $30^\circ\text{N}$  curve in Figure 9 on June 21) is  $14 - 12 = 2$ . So the function is  $L(t) = 12 + 2 \sin\left[\frac{2\pi}{365}(t - 80)\right]$ . March 31 is the 90th day of the year, so the model gives  $L(90) \approx 12.34$  h. The daylight time (5:51 AM to 6:18 PM) is 12 hours and 27 minutes, or 12.45 h. The model value differs from the actual value by  $\frac{12.45 - 12.34}{12.45} \approx 0.009$ , less than 1%.

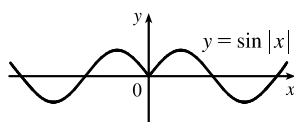
26. Using a sine function to model the brightness of Delta Cephei as a function of time, we take its period to be 5.4 days, its amplitude to be 0.35 (on the scale of magnitude), and its average magnitude to be 4.0. If we take  $t = 0$  at a time of average brightness, then the magnitude (brightness) as a function of time  $t$  in days can be modeled by the formula  $M(t) = 4.0 + 0.35 \sin\left(\frac{2\pi}{5.4}t\right)$ .

27. The water depth  $D(t)$  can be modeled by a cosine function with amplitude  $\frac{12 - 2}{2} = 5$  m, average magnitude  $\frac{12 + 2}{2} = 7$  m, and period 12 hours. High tide occurred at time 6:45 AM ( $t = 6.75$  h), so the curve begins a cycle at time  $t = 6.75$  h (shift 6.75 units to the right). Thus,  $D(t) = 5 \cos\left[\frac{2\pi}{12}(t - 6.75)\right] + 7 = 5 \cos\left[\frac{\pi}{6}(t - 6.75)\right] + 7$ , where  $D$  is in meters and  $t$  is the number of hours after midnight.

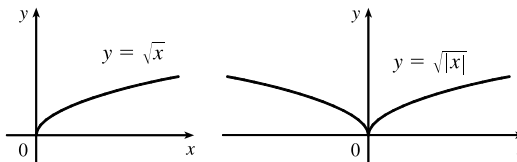
28. The total volume of air  $V(t)$  in the lungs can be modeled by a sine function with amplitude  $\frac{2500 - 2000}{2} = 250$  mL, average volume  $\frac{2500 + 2000}{2} = 2250$  mL, and period 4 seconds. Thus,  $V(t) = 250 \sin\frac{2\pi}{4}t + 2250 = 250 \sin\frac{\pi}{2}t + 2250$ , where  $V$  is in mL and  $t$  is in seconds.

29. (a) To obtain  $y = f(|x|)$ , the portion of the graph of  $y = f(x)$  to the right of the  $y$ -axis is reflected about the  $y$ -axis.

(b)  $y = \sin|x|$

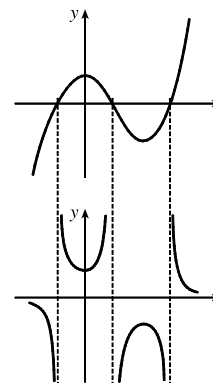


(c)  $y = \sqrt{|x|}$





30. The most important features of the given graph are the  $x$ -intercepts and the maximum and minimum points. The graph of  $y = 1/f(x)$  has vertical asymptotes at the  $x$ -values where there are  $x$ -intercepts on the graph of  $y = f(x)$ . The maximum of 1 on the graph of  $y = f(x)$  corresponds to a minimum of  $1/1 = 1$  on  $y = 1/f(x)$ . Similarly, the minimum on the graph of  $y = f(x)$  corresponds to a maximum on the graph of  $y = 1/f(x)$ . As the values of  $y$  get large (positively or negatively) on the graph of  $y = f(x)$ , the values of  $y$  get close to zero on the graph of  $y = 1/f(x)$ .



31.  $f(x) = x^3 + 2x^2$ ;  $g(x) = 3x^2 - 1$ .  $D = \mathbb{R}$  for both  $f$  and  $g$ .

(a)  $(f + g)(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1$ ,  $D = (-\infty, \infty)$ , or  $\mathbb{R}$ .

(b)  $(f - g)(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1$ ,  $D = \mathbb{R}$ .

(c)  $(fg)(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2$ ,  $D = \mathbb{R}$ .

(d)  $\left(\frac{f}{g}\right)(x) = \frac{x^3 + 2x^2}{3x^2 - 1}$ ,  $D = \left\{x \mid x \neq \pm \frac{1}{\sqrt{3}}\right\}$  since  $3x^2 - 1 \neq 0$ .

32.  $f(x) = \sqrt{3-x}$ ,  $D = (-\infty, 3]$ ;  $g(x) = \sqrt{x^2-1}$ ,  $D = (-\infty, -1] \cup [1, \infty)$ .

(a)  $(f + g)(x) = \sqrt{3-x} + \sqrt{x^2-1}$ ,  $D = (-\infty, -1] \cup [1, 3]$ , which is the intersection of the domains of  $f$  and  $g$ .

(b)  $(f - g)(x) = \sqrt{3-x} - \sqrt{x^2-1}$ ,  $D = (-\infty, -1] \cup [1, 3]$ .

(c)  $(fg)(x) = \sqrt{3-x} \cdot \sqrt{x^2-1}$ ,  $D = (-\infty, -1] \cup [1, 3]$ .

(d)  $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$ ,  $D = (-\infty, -1] \cup (1, 3]$ . We must exclude  $x = \pm 1$  since these values would make  $\frac{f}{g}$  undefined.

33.  $f(x) = 3x + 5$ ;  $g(x) = x^2 + x$ .  $D = \mathbb{R}$  for both  $f$  and  $g$ , and hence for their composites.

(a)  $(f \circ g)(x) = f(g(x)) = f(x^2 + x) = 3(x^2 + x) + 5 = 3x^2 + 3x + 5$ ,  $D = \mathbb{R}$ .

(b)  $(g \circ f)(x) = g(f(x)) = g(3x + 5) = (3x + 5)^2 + (3x + 5)$   
 $= 9x^2 + 30x + 25 + 3x + 5 = 9x^2 + 33x + 30$ ,  $D = \mathbb{R}$ .

(c)  $(f \circ f)(x) = f(f(x)) = f(3x + 5) = 3(3x + 5) + 5 = 9x + 15 + 5 = 9x + 20$ ,  $D = \mathbb{R}$ .

(d)  $(g \circ g)(x) = g(g(x)) = g(x^2 + x) = (x^2 + x)^2 + (x^2 + x)$   
 $= x^4 + 2x^3 + x^2 + x^2 + x = x^4 + 2x^3 + 2x^2 + x$ ,  $D = \mathbb{R}$ .

34.  $f(x) = x^3 - 2$ ;  $g(x) = 1 - 4x$ .  $D = \mathbb{R}$  for both  $f$  and  $g$ , and hence for their composites.

(a)  $(f \circ g)(x) = f(g(x)) = f(1 - 4x) = (1 - 4x)^3 - 2$   
 $= (1)^3 - 3(1)^2(4x) + 3(1)(4x)^2 - (4x)^3 - 2 = 1 - 12x + 48x^2 - 64x^3 - 2$   
 $= -1 - 12x + 48x^2 - 64x^3$ ,  $D = \mathbb{R}$ .

(b)  $(g \circ f)(x) = g(f(x)) = g(x^3 - 2) = 1 - 4(x^3 - 2) = 1 - 4x^3 + 8 = 9 - 4x^3$ ,  $D = \mathbb{R}$ .

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(c)  $(f \circ f)(x) = f(f(x)) = f(x^3 - 2) = (x^3 - 2)^3 - 2$   
 $= (x^3)^3 - 3(x^3)^2(2) + 3(x^3)(2)^2 - (2)^3 - 2 = x^9 - 6x^6 + 12x^3 - 10, D = \mathbb{R}.$

(d)  $(g \circ g)(x) = g(g(x)) = g(1 - 4x) = 1 - 4(1 - 4x) = 1 - 4 + 16x = -3 + 16x, D = \mathbb{R}.$

35.  $f(x) = \sqrt{x+1}, D = \{x \mid x \geq -1\}; g(x) = 4x - 3, D = \mathbb{R}.$

(a)  $(f \circ g)(x) = f(g(x)) = f(4x - 3) = \sqrt{(4x - 3) + 1} = \sqrt{4x - 2}$

The domain of  $f \circ g$  is  $\{x \mid 4x - 3 \geq -1\} = \{x \mid 4x \geq 2\} = \{x \mid x \geq \frac{1}{2}\} = [\frac{1}{2}, \infty).$

(b)  $(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = 4\sqrt{x+1} - 3$

The domain of  $g \circ f$  is  $\{x \mid x \text{ is in the domain of } f \text{ and } f(x) \text{ is in the domain of } g\}$ . This is the domain of  $f$ , that is,  $\{x \mid x + 1 \geq 0\} = \{x \mid x \geq -1\} = [-1, \infty).$

(c)  $(f \circ f)(x) = f(f(x)) = f(\sqrt{x+1}) = \sqrt{\sqrt{x+1} + 1}$

For the domain, we need  $x + 1 \geq 0$ , which is equivalent to  $x \geq -1$ , and  $\sqrt{x+1} \geq -1$ , which is true for all real values of  $x$ . Thus, the domain of  $f \circ f$  is  $[-1, \infty).$

(d)  $(g \circ g)(x) = g(g(x)) = g(4x - 3) = 4(4x - 3) - 3 = 16x - 12 - 3 = 16x - 15, D = \mathbb{R}.$

36.  $f(x) = \sin x; g(x) = x^2 + 1. D = \mathbb{R}$  for both  $f$  and  $g$ , and hence for their composites.

(a)  $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sin(x^2 + 1), D = \mathbb{R}.$

(b)  $(g \circ f)(x) = g(f(x)) = g(\sin x) = (\sin x)^2 + 1 = \sin^2 x + 1, D = \mathbb{R}.$

(c)  $(f \circ f)(x) = f(f(x)) = f(\sin x) = \sin(\sin x), D = \mathbb{R}.$

(d)  $(g \circ g)(x) = g(g(x)) = g(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 1 + 1 = x^4 + 2x^2 + 2, D = \mathbb{R}.$

37.  $f(x) = x + \frac{1}{x}, D = \{x \mid x \neq 0\}; g(x) = \frac{x+1}{x+2}, D = \{x \mid x \neq -2\}$

(a)  $(f \circ g)(x) = f(g(x)) = f\left(\frac{x+1}{x+2}\right) = \frac{x+1}{x+2} + \frac{1}{\frac{x+1}{x+2}} = \frac{x+1}{x+2} + \frac{x+2}{x+1}$   
 $= \frac{(x+1)(x+1) + (x+2)(x+2)}{(x+2)(x+1)} = \frac{(x^2 + 2x + 1) + (x^2 + 4x + 4)}{(x+2)(x+1)} = \frac{2x^2 + 6x + 5}{(x+2)(x+1)}$

Since  $g(x)$  is not defined for  $x = -2$  and  $f(g(x))$  is not defined for  $x = -2$  and  $x = -1$ ,

the domain of  $(f \circ g)(x)$  is  $D = \{x \mid x \neq -2, -1\}.$

(b)  $(g \circ f)(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = \frac{\left(x + \frac{1}{x}\right) + 1}{\left(x + \frac{1}{x}\right) + 2} = \frac{\frac{x^2 + 1 + x}{x}}{\frac{x^2 + 1 + 2x}{x}} = \frac{x^2 + x + 1}{x^2 + 2x + 1} = \frac{x^2 + x + 1}{(x+1)^2}$

Since  $f(x)$  is not defined for  $x = 0$  and  $g(f(x))$  is not defined for  $x = -1$ ,

the domain of  $(g \circ f)(x)$  is  $D = \{x \mid x \neq -1, 0\}.$

$$\begin{aligned}
 \text{(c) } (f \circ f)(x) &= f(f(x)) = f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right) + \frac{1}{x + \frac{1}{x}} = x + \frac{1}{x} + \frac{1}{\frac{x^2+1}{x}} = x + \frac{1}{x} + \frac{x}{x^2+1} \\
 &= \frac{x(x)(x^2+1) + 1(x^2+1) + x(x)}{x(x^2+1)} = \frac{x^4 + x^2 + x^2 + 1 + x^2}{x(x^2+1)} \\
 &= \frac{x^4 + 3x^2 + 1}{x(x^2+1)}, \quad D = \{x \mid x \neq 0\}
 \end{aligned}$$

$$\text{(d) } (g \circ g)(x) = g(g(x)) = g\left(\frac{x+1}{x+2}\right) = \frac{\frac{x+1}{x+2} + 1}{\frac{x+1}{x+2} + 2} = \frac{\frac{x+1+1(x+2)}{x+2}}{\frac{x+1+2(x+2)}{x+2}} = \frac{x+1+x+2}{x+1+2x+4} = \frac{2x+3}{3x+5}$$

Since  $g(x)$  is not defined for  $x = -2$  and  $g(g(x))$  is not defined for  $x = -\frac{5}{3}$ ,

the domain of  $(g \circ g)(x)$  is  $D = \{x \mid x \neq -2, -\frac{5}{3}\}$ .

38.  $f(x) = \frac{x}{1+x}$ ,  $D = \{x \mid x \neq -1\}$ ;  $g(x) = \sin 2x$ ,  $D = \mathbb{R}$ .

(a)  $(f \circ g)(x) = f(g(x)) = f(\sin 2x) = \frac{\sin 2x}{1 + \sin 2x}$

Domain:  $1 + \sin 2x \neq 0 \Rightarrow \sin 2x \neq -1 \Rightarrow 2x \neq \frac{3\pi}{2} + 2\pi n \Rightarrow x \neq \frac{3\pi}{4} + \pi n$  [ $n$  an integer].

(b)  $(g \circ f)(x) = g(f(x)) = g\left(\frac{x}{1+x}\right) = \sin\left(\frac{2x}{1+x}\right)$ .

Domain:  $\{x \mid x \neq -1\}$

(c)  $(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{1+x}\right) = \frac{\frac{x}{1+x}}{1 + \frac{x}{1+x}} = \frac{\left(\frac{x}{1+x}\right) \cdot (1+x)}{\left(1 + \frac{x}{1+x}\right) \cdot (1+x)} = \frac{x}{1+x+x} = \frac{x}{2x+1}$

Since  $f(x)$  is not defined for  $x = -1$ , and  $f(f(x))$  is not defined for  $x = -\frac{1}{2}$ ,

the domain of  $(f \circ f)(x)$  is  $D = \{x \mid x \neq -1, -\frac{1}{2}\}$ .

(d)  $(g \circ g)(g) = g(g(x)) = g(\sin 2x) = \sin(2 \sin 2x)$ .

Domain:  $\mathbb{R}$

39.  $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(\sin(x^2)) = 3 \sin(x^2) - 2$

40.  $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(2\sqrt{x}) = |2\sqrt{x} - 4|$

41.  $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^3 + 2)) = f[(x^3 + 2)^2]$   
 $= f(x^6 + 4x^3 + 4) = \sqrt{(x^6 + 4x^3 + 4) - 3} = \sqrt{x^6 + 4x^3 + 1}$

42.  $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt[3]{x})) = f\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) = \tan\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

43. Let  $g(x) = 2x + x^2$  and  $f(x) = x^4$ . Then  $(f \circ g)(x) = f(g(x)) = f(2x + x^2) = (2x + x^2)^4 = F(x)$ .

44. Let  $g(x) = \cos x$  and  $f(x) = x^2$ . Then  $(f \circ g)(x) = f(g(x)) = f(\cos x) = (\cos x)^2 = \cos^2 x = F(x)$ .

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45. Let  $g(x) = \sqrt[3]{x}$  and  $f(x) = \frac{x}{1+x}$ . Then  $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x}) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}} = F(x)$ .

46. Let  $g(x) = \frac{x}{1+x}$  and  $f(x) = \sqrt[3]{x}$ . Then  $(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{1+x}\right) = \sqrt[3]{\frac{x}{1+x}} = G(x)$ .

47. Let  $g(t) = t^2$  and  $f(t) = \sec t \tan t$ . Then  $(f \circ g)(t) = f(g(t)) = f(t^2) = \sec(t^2) \tan(t^2) = v(t)$ .

48. Let  $g(t) = \tan t$  and  $f(t) = \frac{t}{1+t}$ . Then  $(f \circ g)(t) = f(g(t)) = f(\tan t) = \frac{\tan t}{1 + \tan t} = u(t)$ .

49. Let  $h(x) = \sqrt{x}$ ,  $g(x) = x - 1$ , and  $f(x) = \sqrt{x}$ . Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(\sqrt{x} - 1) = \sqrt{\sqrt{x} - 1} = R(x).$$

50. Let  $h(x) = |x|$ ,  $g(x) = 2 + x$ , and  $f(x) = \sqrt[5]{x}$ . Then

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f(2 + |x|) = \sqrt[5]{2 + |x|} = H(x).$$

51. Let  $h(t) = \cos t$ ,  $g(t) = \sin t$ , and  $f(t) = t^2$ . Then

$$(f \circ g \circ h)(t) = f(g(h(t))) = f(g(\cos t)) = f(\sin(\cos t)) = [\sin(\cos t)]^2 = \sin^2(\cos t) = S(t).$$

52. (a)  $f(g(1)) = f(6) = 5$

(b)  $g(f(1)) = g(3) = 2$

(c)  $f(f(1)) = f(3) = 4$

(d)  $g(g(1)) = g(6) = 3$

(e)  $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f)  $(f \circ g)(6) = f(g(6)) = f(3) = 4$

53. (a)  $g(2) = 5$ , because the point  $(2, 5)$  is on the graph of  $g$ . Thus,  $f(g(2)) = f(5) = 4$ , because the point  $(5, 4)$  is on the graph of  $f$ .

(b)  $g(f(0)) = g(0) = 3$

(c)  $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d)  $(g \circ f)(6) = g(f(6)) = g(6)$ . This value is not defined, because there is no point on the graph of  $g$  that has  $x$ -coordinate 6.

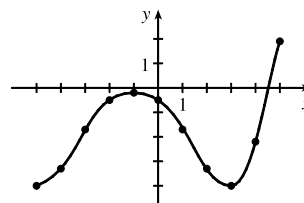
(e)  $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f)  $(f \circ f)(4) = f(f(4)) = f(2) = -2$

54. To find a particular value of  $f(g(x))$ , say for  $x = 0$ , we note from the graph that  $g(0) \approx 2.8$  and  $f(2.8) \approx -0.5$ . Thus,  $f(g(0)) \approx f(2.8) \approx -0.5$ . The other values listed in the table were obtained in a similar fashion.

$x$	$g(x)$	$f(g(x))$
-5	-0.2	-4
-4	1.2	-3.3
-3	2.2	-1.7
-2	2.8	-0.5
-1	3	-0.2

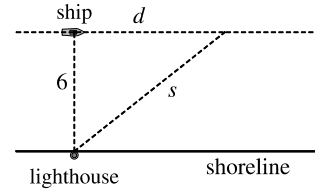
$x$	$g(x)$	$f(g(x))$
0	2.8	-0.5
1	2.2	-1.7
2	1.2	-3.3
3	-0.2	-4
4	-1.9	-2.2
5	-4.1	1.9



55. (a) Using the relationship  $distance = rate \cdot time$  with the radius  $r$  as the distance, we have  $r(t) = 60t$ .  
 (b)  $A = \pi r^2 \Rightarrow (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$ . This formula gives us the extent of the rippled area (in  $cm^2$ ) at any time  $t$ .

56. (a) The radius  $r$  of the balloon is increasing at a rate of 2 cm/s, so  $r(t) = (2 \text{ cm/s})(t \text{ s}) = 2t$  (in cm).  
 (b) Using  $V = \frac{4}{3}\pi r^3$ , we get  $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$ .  
 The result,  $V = \frac{32}{3}\pi t^3$ , gives the volume of the balloon (in  $cm^3$ ) as a function of time (in s).

57. (a) From the figure, we have a right triangle with legs 6 and  $d$ , and hypotenuse  $s$ .  
 By the Pythagorean Theorem,  $d^2 + 6^2 = s^2 \Rightarrow s = f(d) = \sqrt{d^2 + 36}$ .



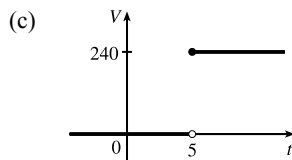
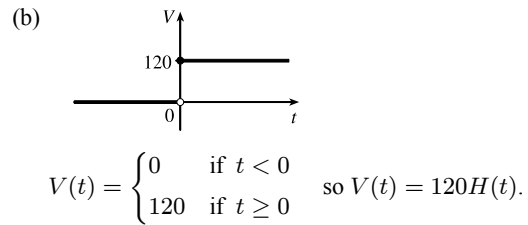
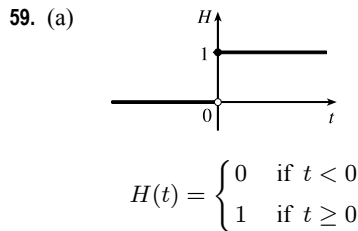
- (b) Using  $d = rt$ , we get  $d = (30 \text{ km/h})(t \text{ hours}) = 30t$  (in km). Thus,  
 $d = g(t) = 30t$ .

- (c)  $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$ . This function represents the distance between the lighthouse and the ship as a function of the time elapsed since noon.

58. (a)  $d = rt \Rightarrow d(t) = 350t$

- (b) There is a Pythagorean relationship involving the legs with lengths  $d$  and 1 and the hypotenuse with length  $s$ :  
 $d^2 + 1^2 = s^2$ . Thus,  $s(d) = \sqrt{d^2 + 1}$ .

- (c)  $(s \circ d)(t) = s(d(t)) = s(350t) = \sqrt{(350t)^2 + 1}$



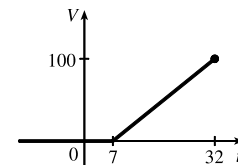
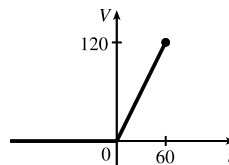
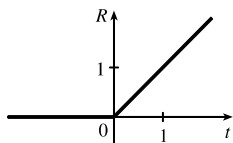
Starting with the formula in part (b), we replace 120 with 240 to reflect the different voltage. Also, because we are starting 5 units to the right of  $t = 0$ , we replace  $t$  with  $t - 5$ . Thus, the formula is  $V(t) = 240H(t - 5)$ .

60. (a)  $R(t) = tH(t)$   

$$= \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}$$

- (b)  $V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 2t & \text{if } 0 \leq t \leq 60 \end{cases}$   
 so  $V(t) = 2tH(t), t \leq 60$ .

- (c)  $V(t) = \begin{cases} 0 & \text{if } t < 7 \\ 4(t - 7) & \text{if } 7 \leq t \leq 32 \end{cases}$   
 so  $V(t) = 4(t - 7)H(t - 7), t \leq 32$ .



61. If  $f(x) = m_1x + b_1$  and  $g(x) = m_2x + b_2$ , then

$$(f \circ g)(x) = f(g(x)) = f(m_2x + b_2) = m_1(m_2x + b_2) + b_1 = m_1m_2x + m_1b_2 + b_1.$$

So  $f \circ g$  is a linear function with slope  $m_1m_2$ .

62. If  $A(x) = 1.04x$ , then

$$(A \circ A)(x) = A(A(x)) = A(1.04x) = 1.04(1.04x) = (1.04)^2x,$$

$$(A \circ A \circ A)(x) = A((A \circ A)(x)) = A((1.04)^2x) = 1.04(1.04)^2x = (1.04)^3x, \text{ and}$$

$$(A \circ A \circ A \circ A)(x) = A((A \circ A \circ A)(x)) = A((1.04)^3x) = 1.04(1.04)^3x = (1.04)^4x.$$

These compositions represent the amount of the investment after 2, 3, and 4 years.

Based on this pattern, when we compose  $n$  copies of  $A$ , we get the formula  $\underbrace{(A \circ A \circ \dots \circ A)}_{n \text{ A's}}(x) = (1.04)^n x$ .

63. (a) By examining the variable terms in  $g$  and  $h$ , we deduce that we must square  $g$  to get the terms  $4x^2$  and  $4x$  in  $h$ . If we let

$$f(x) = x^2 + c, \text{ then } (f \circ g)(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2 + c = 4x^2 + 4x + (1 + c). \text{ Since}$$

$$h(x) = 4x^2 + 4x + 7, \text{ we must have } 1 + c = 7. \text{ So } c = 6 \text{ and } f(x) = x^2 + 6.$$

(b) We need a function  $g$  so that  $f(g(x)) = 3(g(x)) + 5 = h(x)$ . But

$$h(x) = 3x^2 + 3x + 2 = 3(x^2 + x) + 2 = 3(x^2 + x - 1) + 5, \text{ so we see that } g(x) = x^2 + x - 1.$$

64. We need a function  $g$  so that  $g(f(x)) = g(x + 4) = h(x) = 4x - 1 = 4(x + 4) - 17$ . So we see that the function  $g$  must be

$$g(x) = 4x - 17.$$

65. We need to examine  $h(-x)$ .

$$h(-x) = (f \circ g)(-x) = f(g(-x)) = f(g(x)) \quad [\text{because } g \text{ is even}] = h(x)$$

Because  $h(-x) = h(x)$ ,  $h$  is an even function.

66.  $h(-x) = f(g(-x)) = f(-g(x))$ . At this point, we can't simplify the expression, so we might try to find a counterexample to show that  $h$  is not an odd function. Let  $g(x) = x$ , an odd function, and  $f(x) = x^2 + x$ . Then  $h(x) = x^2 + x$ , which is neither even nor odd.

Now suppose  $f$  is an odd function. Then  $f(-g(x)) = -f(g(x)) = -h(x)$ . Hence,  $h(-x) = -h(x)$ , and so  $h$  is odd if both  $f$  and  $g$  are odd.

Now suppose  $f$  is an even function. Then  $f(-g(x)) = f(g(x)) = h(x)$ . Hence,  $h(-x) = h(x)$ , and so  $h$  is even if  $g$  is odd and  $f$  is even.

## 1.4 Exponential Functions

1. (a)  $\frac{4^{-3}}{2^{-8}} = \frac{2^8}{4^3} = \frac{2^8}{(2^2)^3} = \frac{2^8}{2^6} = 2^{8-6} = 2^2 = 4$

(b)  $\frac{1}{\sqrt[3]{x^4}} = \frac{1}{x^{4/3}} = x^{-4/3}$

2. (a)  $8^{4/3} = (8^{1/3})^4 = 2^4 = 16$

(b)  $x(3x^2)^3 = x \cdot 3^3(x^2)^3 = 27x \cdot x^6 = 27x^7$

3. (a)  $b^8(2b)^4 = b^8 \cdot 2^4b^4 = 16b^{12}$

(b)  $\frac{(6y^3)^4}{2y^5} = \frac{6^4(y^3)^4}{2y^5} = \frac{1296y^{12}}{2y^5} = 648y^7$

4. (a)  $\frac{x^{2n} \cdot x^{3n-1}}{x^{n+2}} = \frac{x^{2n+3n-1}}{x^{n+2}} = \frac{x^{5n-1}}{x^{n+2}} = x^{4n-3}$

(b)  $\frac{\sqrt{a}\sqrt{b}}{\sqrt[3]{ab}} = \frac{\sqrt{a}\sqrt[3]{b}}{\sqrt[3]{a}\sqrt[3]{b}} = \frac{a^{1/2}b^{1/4}}{a^{1/3}b^{1/3}} = a^{(1/2-1/3)}b^{(1/4-1/3)} = a^{1/6}b^{-1/12}$

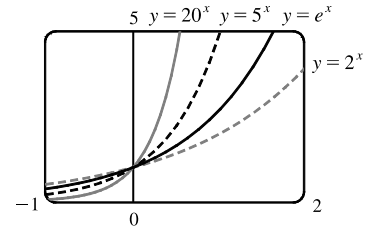
5. (a)  $f(x) = b^x$ ,  $b > 0$       (b)  $\mathbb{R}$       (c)  $(0, \infty)$       (d) See Figures 4(c), 4(b), and 4(a), respectively.

6. (a) The number  $e$  is the value of  $a$  such that the slope of the tangent line at  $x = 0$  on the graph of  $y = a^x$  is exactly 1.

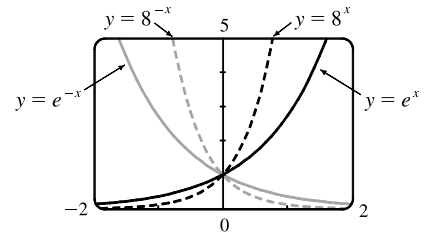
(b)  $e \approx 2.71828$       (c)  $f(x) = e^x$

7. All of these graphs approach 0 as  $x \rightarrow -\infty$ , all of them pass through the point  $(0, 1)$ , and all of them are increasing and approach  $\infty$  as  $x \rightarrow \infty$ . The larger the base, the faster the function increases for  $x > 0$ , and the faster it approaches 0 as  $x \rightarrow -\infty$ .

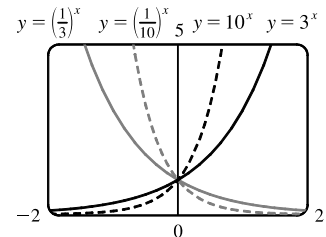
*Note:* The notation “ $x \rightarrow \infty$ ” can be thought of as “ $x$  becomes large” at this point. More details on this notation are given in Chapter 2.



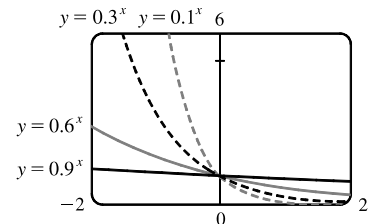
8. The graph of  $e^{-x}$  is the reflection of the graph of  $e^x$  about the  $y$ -axis, and the graph of  $8^{-x}$  is the reflection of that of  $8^x$  about the  $y$ -axis. The graph of  $8^{-x}$  increases more quickly than that of  $e^x$  for  $x > 0$ , and approaches 0 faster as  $x \rightarrow -\infty$ .



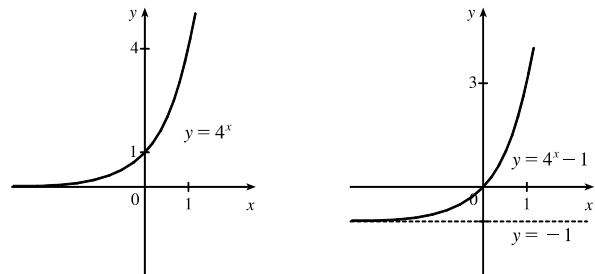
9. The functions with bases greater than 1 ( $3^x$  and  $10^x$ ) are increasing, while those with bases less than 1 [ $(\frac{1}{3})^x$  and  $(\frac{1}{10})^x$ ] are decreasing. The graph of  $(\frac{1}{3})^x$  is the reflection of that of  $3^x$  about the  $y$ -axis, and the graph of  $(\frac{1}{10})^x$  is the reflection of that of  $10^x$  about the  $y$ -axis. The graph of  $10^x$  increases more quickly than that of  $3^x$  for  $x > 0$ , and approaches 0 faster as  $x \rightarrow -\infty$ .



10. Each of the graphs approaches  $\infty$  as  $x \rightarrow -\infty$ , and each approaches 0 as  $x \rightarrow \infty$ . The smaller the base, the faster the function grows as  $x \rightarrow -\infty$ , and the faster it approaches 0 as  $x \rightarrow \infty$ .

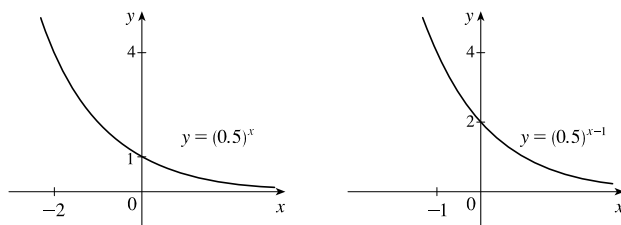


11. We start with the graph of  $y = 4^x$  (Figure 3) and shift it 1 unit down to obtain the graph of  $y = 4^x - 1$ .

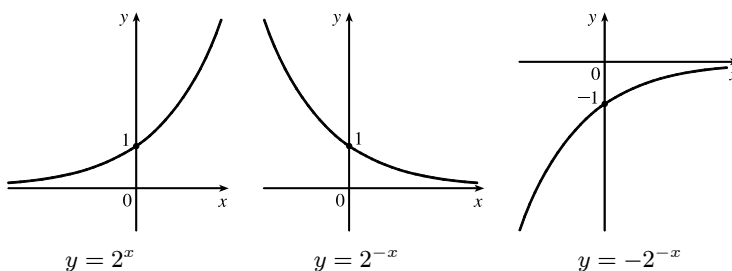


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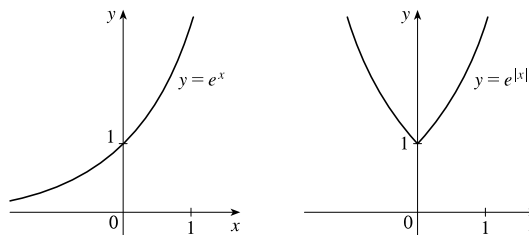
12. We start with the graph of  $y = (0.5)^x$  (Figure 3) and shift it 1 unit to the right to obtain the graph of  $y = (0.5)^{x-1}$ .



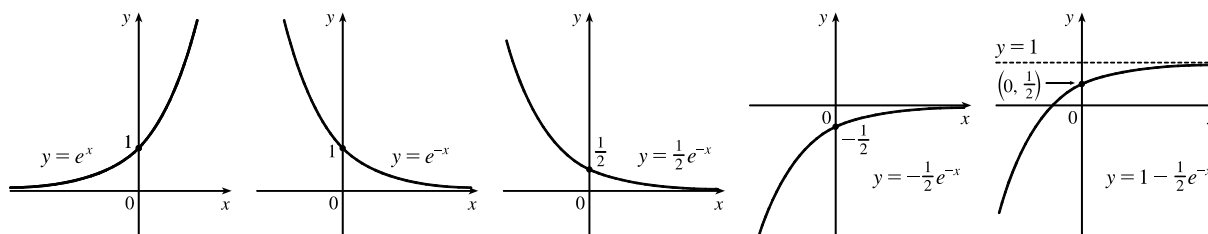
13. We start with the graph of  $y = 2^x$  (Figure 16), reflect it about the  $y$ -axis, and then about the  $x$ -axis (or just rotate  $180^\circ$  to handle both reflections) to obtain the graph of  $y = -2^{-x}$ . In each graph,  $y = 0$  is the horizontal asymptote.



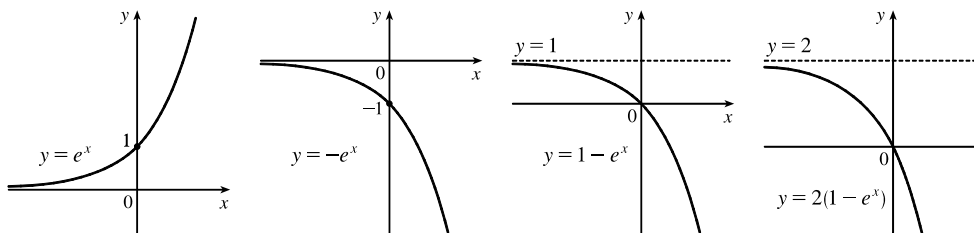
14. We start with the graph of  $y = e^x$  (Figure 16) and reflect the portion of the graph in the first quadrant about the  $y$ -axis to obtain the graph of  $y = e^{|x|}$ .



15. We start with the graph of  $y = e^x$  (Figure 16) and reflect about the  $y$ -axis to get the graph of  $y = e^{-x}$ . Then we compress the graph vertically by a factor of 2 to obtain the graph of  $y = \frac{1}{2}e^{-x}$  and then reflect about the  $x$ -axis to get the graph of  $y = -\frac{1}{2}e^{-x}$ . Finally, we shift the graph upward one unit to get the graph of  $y = 1 - \frac{1}{2}e^{-x}$ .



16. We start with the graph of  $y = e^x$  (Figure 13) and reflect about the  $x$ -axis to get the graph of  $y = -e^x$ . Then shift the graph upward one unit to get the graph of  $y = 1 - e^x$ . Finally, we stretch the graph vertically by a factor of 2 to obtain the graph of  $y = 2(1 - e^x)$ .

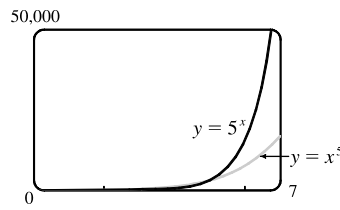
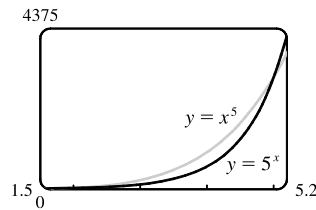
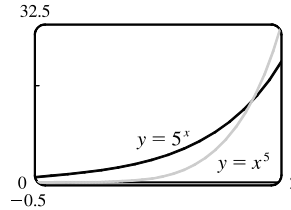
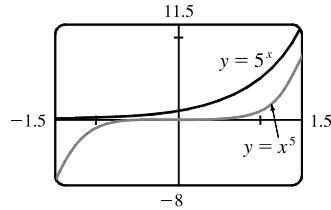




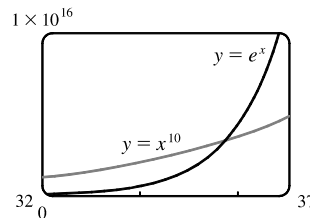
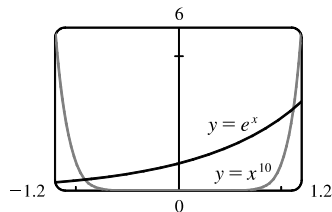
17. (a) To find the equation of the graph that results from shifting the graph of  $y = e^x$  2 units downward, we subtract 2 from the original function to get  $y = e^x - 2$ .
- (b) To find the equation of the graph that results from shifting the graph of  $y = e^x$  2 units to the right, we replace  $x$  with  $x - 2$  in the original function to get  $y = e^{(x-2)}$ .
- (c) To find the equation of the graph that results from reflecting the graph of  $y = e^x$  about the  $x$ -axis, we multiply the original function by  $-1$  to get  $y = -e^x$ .
- (d) To find the equation of the graph that results from reflecting the graph of  $y = e^x$  about the  $y$ -axis, we replace  $x$  with  $-x$  in the original function to get  $y = e^{-x}$ .
- (e) To find the equation of the graph that results from reflecting the graph of  $y = e^x$  about the  $x$ -axis and then about the  $y$ -axis, we first multiply the original function by  $-1$  (to get  $y = -e^x$ ) and then replace  $x$  with  $-x$  in this equation to get  $y = -e^{-x}$ .
18. (a) This reflection consists of first reflecting the graph about the  $x$ -axis (giving the graph with equation  $y = -e^x$ ) and then shifting this graph  $2 \cdot 4 = 8$  units upward. So the equation is  $y = -e^x + 8$ .
- (b) This reflection consists of first reflecting the graph about the  $y$ -axis (giving the graph with equation  $y = e^{-x}$ ) and then shifting this graph  $2 \cdot 2 = 4$  units to the right. So the equation is  $y = e^{-(x-4)}$ .
19. (a) The denominator is zero when  $1 - e^{1-x^2} = 0 \Leftrightarrow e^{1-x^2} = 1 \Leftrightarrow 1 - x^2 = 0 \Leftrightarrow x = \pm 1$ . Thus, the function  $f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$  has domain  $\{x \mid x \neq \pm 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ .
- (b) The denominator is never equal to zero, so the function  $f(x) = \frac{1+x}{e^{\cos x}}$  has domain  $\mathbb{R}$ , or  $(-\infty, \infty)$ .
20. (a) The function  $g(t) = \sqrt{10^t - 100}$  has domain  $\{t \mid 10^t - 100 \geq 0\} = \{t \mid 10^t \geq 10^2\} = \{t \mid t \geq 2\} = [2, \infty)$ .
- (b) The sine and exponential functions have domain  $\mathbb{R}$ , so  $g(t) = \sin(e^t - 1)$  also has domain  $\mathbb{R}$ .
21. Use  $y = Cb^x$  with the points  $(1, 6)$  and  $(3, 24)$ .  $6 = Cb^1$  [ $C = \frac{6}{b}$ ] and  $24 = Cb^3 \Rightarrow 24 = \left(\frac{6}{b}\right)b^3 \Rightarrow 4 = b^2 \Rightarrow b = 2$  [since  $b > 0$ ] and  $C = \frac{6}{2} = 3$ . The function is  $f(x) = 3 \cdot 2^x$ .
22. Use  $y = Cb^x$  with the points  $(-1, 3)$  and  $(1, \frac{4}{3})$ . From the point  $(-1, 3)$ , we have  $3 = Cb^{-1}$ , hence  $C = 3b$ . Using this and the point  $(1, \frac{4}{3})$ , we get  $\frac{4}{3} = Cb^1 \Rightarrow \frac{4}{3} = (3b)b \Rightarrow \frac{4}{9} = b^2 \Rightarrow b = \frac{2}{3}$  [since  $b > 0$ ] and  $C = 3\left(\frac{2}{3}\right) = 2$ . The function is  $f(x) = 2\left(\frac{2}{3}\right)^x$ .
23. If  $f(x) = 5^x$ , then  $\frac{f(x+h) - f(x)}{h} = \frac{5^{x+h} - 5^x}{h} = \frac{5^x 5^h - 5^x}{h} = \frac{5^x(5^h - 1)}{h} = 5^x \left(\frac{5^h - 1}{h}\right)$ .
24. Suppose the month is February. Your payment on the 28th day would be  $2^{28-1} = 2^{27} = 134,217,728$  cents, or \$1,342,177.28. Clearly, the second method of payment results in a larger amount for any month.
25.  $2 \text{ ft} = 24 \text{ in}$ ,  $f(24) = 24^2 \text{ in} = 576 \text{ in} = 48 \text{ ft}$ .  $g(24) = 2^{24} \text{ in} = 2^{24}/(12 \cdot 5280) \text{ mi} \approx 265 \text{ mi}$

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26. We see from the graphs that for  $x$  less than about 1.8,  $g(x) = 5^x > f(x) = x^5$ , and then near the point (1.8, 17.1) the curves intersect. Then  $f(x) > g(x)$  from  $x \approx 1.8$  until  $x = 5$ . At (5, 3125) there is another point of intersection, and for  $x > 5$  we see that  $g(x) > f(x)$ . In fact,  $g$  increases much more rapidly than  $f$  beyond that point.

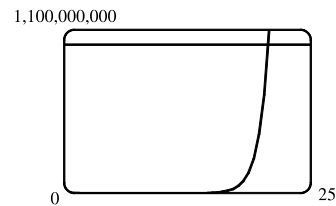


27. The graph of  $g$  finally surpasses that of  $f$  at  $x \approx 35.8$ .

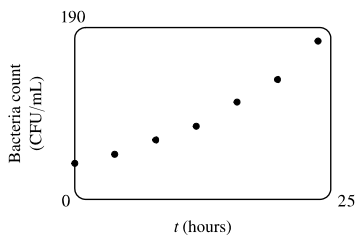


28. We graph  $y = e^x$  and  $y = 1,000,000,000$  and determine where

$e^x = 1 \times 10^9$ . This seems to be true at  $x \approx 20.723$ , so  $e^x > 1 \times 10^9$  for  $x > 20.723$ .



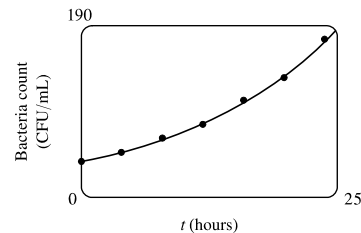
29. (a)



(b) Using a graphing calculator, we obtain the exponential

curve  $f(t) = 36.89301(1.06614)^t$ .

(c) Using the TRACE and zooming in, we find that the bacteria count doubles from 37 to 74 in about 10.87 hours.



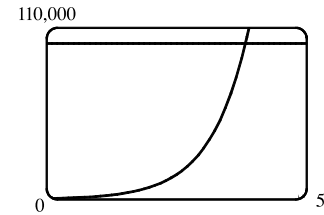
30. (a) Three hours represents 6 doubling periods (one doubling period is 30 minutes).  $500 \cdot 2^6 = 32,000$

(b) In  $t$  hours, there will be  $2t$  doubling periods. The initial population is 500,

so the population  $y$  at time  $t$  is  $y = 500 \cdot 2^{2t}$ .

(c)  $t = \frac{40}{60} = \frac{2}{3} \Rightarrow y = 500 \cdot 2^{2(2/3)} \approx 1260$

(d) We graph  $y_1 = 500 \cdot 2^{2t}$  and  $y_2 = 100,000$ . The two curves intersect at  $t \approx 3.82$ , so the population reaches 100,000 in about 3.82 hours.



31. (a) Fifteen days represents 3 half-life periods (one half-life period is 5 days).  $200 \left(\frac{1}{2}\right)^3 = 25$  mg

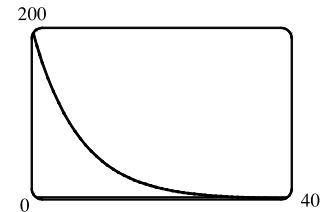
(b) In  $t$  hours, there will be  $t/5$  half-life periods. The initial amount is 200 mg,

so the amount remaining after  $t$  days is  $y = 200 \left(\frac{1}{2}\right)^{t/5}$  mg, or equivalently,

$y = 200 \cdot 2^{-t/5}$  mg.

(c)  $t = 3$  weeks = 21 days  $\Rightarrow y = 200 \cdot 2^{-21/5} \approx 10.9$  mg

(d) We graph  $y_1 = 200 \cdot 2^{-t/5}$  and  $y_2 = 1$ . The two curves intersect at  $t \approx 38.2$ , so the mass will be reduced to 1 mg in about 38.2 days.



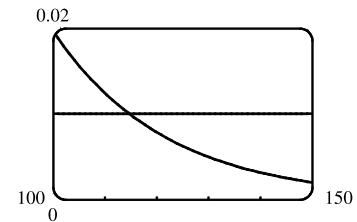
32. (a) Sixty hours represents 4 half-life periods.  $2 \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{8}$  g

(b) In  $t$  hours, there will be  $t/15$  half-life periods. The initial mass is 2 g,

so the mass  $y$  at time  $t$  is  $y = 2 \cdot \left(\frac{1}{2}\right)^{t/15}$ .

(c) 4 days =  $4 \cdot 24 = 96$  hours.  $t = 96 \Rightarrow y = 2 \cdot \left(\frac{1}{2}\right)^{96/15} \approx 0.024$  g

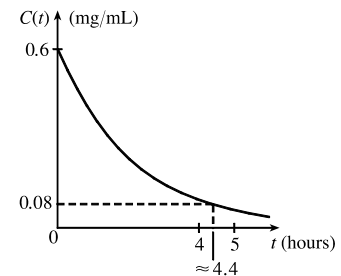
(d)  $y = 0.01 \Rightarrow t \approx 114.7$  hours



33. From the table, we see that  $V(1) = 76$ . In Figure 11, we estimate that  $V = 38$  (half of 76) when  $t \approx 4.5$ . This gives us a half-life of  $4.5 - 1 = 3.5$  days.

34. (a) The exponential decay model has the form  $C(t) = a\left(\frac{1}{2}\right)^{t/1.5}$ , where  $t$  is the number of hours after midnight and  $C(t)$  is the BAC. When  $t = 0$ ,  $C(t) = 0.6$ , so  $0.6 = a\left(\frac{1}{2}\right)^0 \Leftrightarrow a = 0.6$ . Thus, the model is  $C(t) = 0.6\left(\frac{1}{2}\right)^{t/1.5}$ .

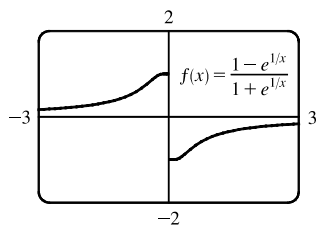
(b) From the graph, we estimate that the BAC is 0.08 mg/mL when  $t \approx 4.4$  hours. (Note that the legal limit is often 0.08%, which is not 0.08 mg/mL.)



35. Let  $t = 0$  correspond to 1950 to get the model  $P = ab^t$ , where  $a \approx 2614.086$  and  $b \approx 1.01693$ . To estimate the population in 1993, let  $t = 43$  to obtain  $P \approx 5381$  million. To predict the population in 2020, let  $t = 70$  to obtain  $P \approx 8466$  million.

36. Let  $t = 0$  correspond to 1900 to get the model  $P = ab^t$ , where  $a \approx 80.8498$  and  $b \approx 1.01269$ . To estimate the population in 1925, let  $t = 25$  to obtain  $P \approx 111$  million. To predict the population in 2020, let  $t = 120$  to obtain  $P \approx 367$  million.

37.



From the graph, it appears that  $f$  is an odd function ( $f$  is undefined for  $x = 0$ ).

To prove this, we must show that  $f(-x) = -f(x)$ .

$$\begin{aligned} f(-x) &= \frac{1 - e^{1/(-x)}}{1 + e^{1/(-x)}} = \frac{1 - e^{(-1/x)}}{1 + e^{(-1/x)}} = \frac{1 - \frac{1}{e^{1/x}}}{1 + \frac{1}{e^{1/x}}} \cdot \frac{e^{1/x}}{e^{1/x}} = \frac{e^{1/x} - 1}{e^{1/x} + 1} \\ &= -\frac{1 - e^{1/x}}{1 + e^{1/x}} = -f(x) \end{aligned}$$

so  $f$  is an odd function.

38. We'll start with  $b = -1$  and graph  $f(x) = \frac{1}{1 + ae^{bx}}$  for  $a = 0.1, 1, \text{ and } 5$ .

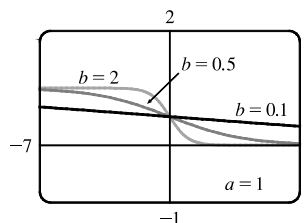
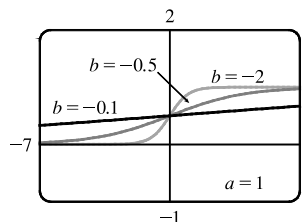
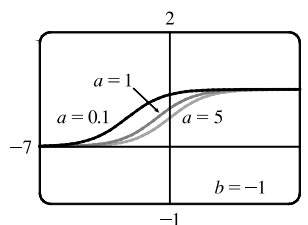
From the graph, we see that there is a horizontal asymptote  $y = 0$  as  $x \rightarrow -\infty$  and a horizontal asymptote  $y = 1$  as  $x \rightarrow \infty$ . If  $a = 1$ , the  $y$ -intercept is  $(0, \frac{1}{2})$ .

As  $a$  gets smaller (close to 0), the graph of  $f$  moves left. As  $a$  gets larger, the graph of  $f$  moves right.

As  $b$  changes from  $-1$  to 0, the graph of  $f$  is stretched horizontally. As  $b$  changes through large negative values, the graph of  $f$  is compressed horizontally. (This takes care of negatives values of  $b$ .)

If  $b$  is positive, the graph of  $f$  is reflected through the  $y$ -axis.

Last, if  $b = 0$ , the graph of  $f$  is the horizontal line  $y = 1/(1 + a)$ .



## 1.5 Inverse Functions and Logarithms

1. (a) See Definition 1.  
(b) It must pass the Horizontal Line Test.
2. (a)  $f^{-1}(y) = x \Leftrightarrow f(x) = y$  for any  $y$  in  $B$ . The domain of  $f^{-1}$  is  $B$  and the range of  $f^{-1}$  is  $A$ .  
(b) See the steps in (5).  
(c) Reflect the graph of  $f$  about the line  $y = x$ .
3.  $f$  is not one-to-one because  $2 \neq 6$ , but  $f(2) = 2.0 = f(6)$ .
4.  $f$  is one-to-one because it never takes on the same value twice.
5. We could draw a horizontal line that intersects the graph in more than one point. Thus, by the Horizontal Line Test, the function is not one-to-one.

6. No horizontal line intersects the graph more than once. Thus, by the Horizontal Line Test, the function is one-to-one.
7. No horizontal line intersects the graph more than once. Thus, by the Horizontal Line Test, the function is one-to-one.
8. We could draw a horizontal line that intersects the graph in more than one point. Thus, by the Horizontal Line Test, the function is not one-to-one.
9. The graph of  $f(x) = 2x - 3$  is a line with slope 2. It passes the Horizontal Line Test, so  $f$  is one-to-one.  
*Algebraic solution:* If  $x_1 \neq x_2$ , then  $2x_1 \neq 2x_2 \Rightarrow 2x_1 - 3 \neq 2x_2 - 3 \Rightarrow f(x_1) \neq f(x_2)$ , so  $f$  is one-to-one.
10. The graph of  $f(x) = x^4 - 16$  is symmetric with respect to the  $y$ -axis. Pick any  $x$ -values equidistant from 0 to find two equal function values. For example,  $f(-1) = -15$  and  $f(1) = -15$ , so  $f$  is not one-to-one.
11.  $g(x) = 1 - \sin x$ .  $g(0) = 1$  and  $g(\pi) = 1$ , so  $g$  is not one-to-one.
12. The graph of  $g(x) = \sqrt[3]{x}$  passes the Horizontal Line Test, so  $g$  is one-to-one.
13. A football will attain every height  $h$  up to its maximum height twice: once on the way up, and again on the way down. Thus, even if  $t_1$  does not equal  $t_2$ ,  $f(t_1)$  may equal  $f(t_2)$ , so  $f$  is not 1-1.
14.  $f$  is not 1-1 because eventually we all stop growing and therefore, there are two times at which we have the same height.
15. (a) Since  $f$  is 1-1,  $f(6) = 17 \Leftrightarrow f^{-1}(17) = 6$ .  
 (b) Since  $f$  is 1-1,  $f^{-1}(3) = 2 \Leftrightarrow f(2) = 3$ .
16. First, we must determine  $x$  such that  $f(x) = 3$ . By inspection, we see that if  $x = 1$ , then  $f(1) = 3$ . Since  $f$  is 1-1 ( $f$  is an increasing function), it has an inverse, and  $f^{-1}(3) = 1$ . If  $f$  is a 1-1 function, then  $f(f^{-1}(a)) = a$ , so  $f(f^{-1}(2)) = 2$ .
17. First, we must determine  $x$  such that  $g(x) = 4$ . By inspection, we see that if  $x = 0$ , then  $g(x) = 4$ . Since  $g$  is 1-1 ( $g$  is an increasing function), it has an inverse, and  $g^{-1}(4) = 0$ .
18. (a)  $f$  is 1-1 because it passes the Horizontal Line Test.  
 (b) Domain of  $f = [-3, 3] = \text{Range of } f^{-1}$ . Range of  $f = [-1, 3] = \text{Domain of } f^{-1}$ .  
 (c) Since  $f(0) = 2$ ,  $f^{-1}(2) = 0$ .  
 (d) Since  $f(-1.7) \approx 0$ ,  $f^{-1}(0) \approx -1.7$ .
19. We solve  $C = \frac{5}{9}(F - 32)$  for  $F$ :  $\frac{9}{5}C = F - 32 \Rightarrow F = \frac{9}{5}C + 32$ . This gives us a formula for the inverse function, that is, the Fahrenheit temperature  $F$  as a function of the Celsius temperature  $C$ .  $F \geq -459.67 \Rightarrow \frac{9}{5}C + 32 \geq -459.67 \Rightarrow \frac{9}{5}C \geq -491.67 \Rightarrow C \geq -273.15$ , the domain of the inverse function.
20.  $m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{m_0^2}{m^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{m_0^2}{m^2} \Rightarrow v^2 = c^2 \left(1 - \frac{m_0^2}{m^2}\right) \Rightarrow v = c \sqrt{1 - \frac{m_0^2}{m^2}}$ .  
 This formula gives us the speed  $v$  of the particle in terms of its mass  $m$ , that is,  $v = f^{-1}(m)$ .
21.  $y = f(x) = 1 + \sqrt{2 + 3x}$  ( $y \geq 1$ )  $\Rightarrow y - 1 = \sqrt{2 + 3x} \Rightarrow (y - 1)^2 = 2 + 3x \Rightarrow (y - 1)^2 - 2 = 3x \Rightarrow x = \frac{1}{3}(y - 1)^2 - \frac{2}{3}$ . Interchange  $x$  and  $y$ :  $y = \frac{1}{3}(x - 1)^2 - \frac{2}{3}$ . So  $f^{-1}(x) = \frac{1}{3}(x - 1)^2 - \frac{2}{3}$ . Note that the domain of  $f^{-1}$  is  $x \geq 1$ .

22.  $y = f(x) = \frac{4x-1}{2x+3} \Rightarrow y(2x+3) = 4x-1 \Rightarrow 2xy+3y = 4x-1 \Rightarrow 3y+1 = 4x-2xy \Rightarrow$

$3y+1 = (4-2y)x \Rightarrow x = \frac{3y+1}{4-2y}$ . Interchange  $x$  and  $y$ :  $y = \frac{3x+1}{4-2x}$ . So  $f^{-1}(x) = \frac{3x+1}{4-2x}$ .

23.  $y = f(x) = e^{2x-1} \Rightarrow \ln y = 2x-1 \Rightarrow 1 + \ln y = 2x \Rightarrow x = \frac{1}{2}(1 + \ln y)$ .

Interchange  $x$  and  $y$ :  $y = \frac{1}{2}(1 + \ln x)$ . So  $f^{-1}(x) = \frac{1}{2}(1 + \ln x)$ .

24.  $y = f(x) = x^2 - x \ (x \geq \frac{1}{2}) \Rightarrow y = x^2 - x + \frac{1}{4} - \frac{1}{4} \Rightarrow y = (x - \frac{1}{2})^2 - \frac{1}{4} \Rightarrow$

$y + \frac{1}{4} = (x - \frac{1}{2})^2 \Rightarrow x - \frac{1}{2} = \sqrt{y + \frac{1}{4}} \Rightarrow x = \frac{1}{2} + \sqrt{y + \frac{1}{4}}$ . Interchange  $x$  and  $y$ :  $y = \frac{1}{2} + \sqrt{x + \frac{1}{4}}$ . So

$f^{-1}(x) = \frac{1}{2} + \sqrt{x + \frac{1}{4}}$ .

25.  $y = f(x) = \ln(x+3) \Rightarrow x+3 = e^y \Rightarrow x = e^y - 3$ . Interchange  $x$  and  $y$ :  $y = e^x - 3$ . So  $f^{-1}(x) = e^x - 3$ .

26.  $y = f(x) = \frac{1-e^{-x}}{1+e^{-x}} \Rightarrow y(1+e^{-x}) = 1-e^{-x} \Rightarrow y + ye^{-x} = 1-e^{-x} \Rightarrow ye^x + y = e^x - 1$  [multiply

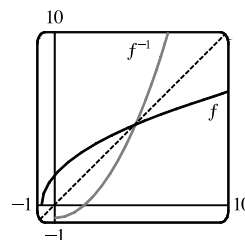
each term by  $e^x$ ]  $\Rightarrow ye^x - e^x = -y - 1 \Rightarrow e^x(y-1) = -y-1 \Rightarrow e^x = \frac{1+y}{1-y} \Rightarrow x = \ln\left(\frac{1+y}{1-y}\right)$ .

Interchange  $x$  and  $y$ :  $y = \ln\left(\frac{1+x}{1-x}\right)$ . So  $f^{-1}(x) = \ln\left(\frac{1+x}{1-x}\right)$ .

27.  $y = f(x) = \sqrt{4x+3} \ (y \geq 0) \Rightarrow y^2 = 4x+3 \Rightarrow x = \frac{y^2-3}{4}$ .

Interchange  $x$  and  $y$ :  $y = \frac{x^2-3}{4}$ . So  $f^{-1}(x) = \frac{x^2-3}{4} \ (x \geq 0)$ . From

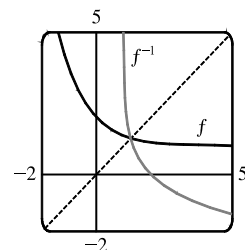
the graph, we see that  $f$  and  $f^{-1}$  are reflections about the line  $y = x$ .



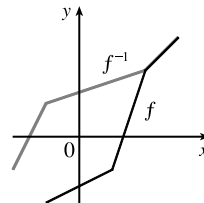
28.  $y = f(x) = 1 + e^{-x} \Rightarrow e^{-x} = y-1 \Rightarrow -x = \ln(y-1) \Rightarrow$

$x = -\ln(y-1)$ . Interchange  $x$  and  $y$ :  $y = -\ln(x-1)$ .

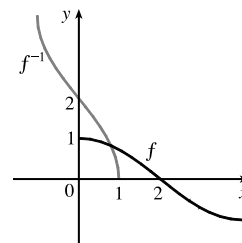
So  $f^{-1}(x) = -\ln(x-1)$ . From the graph, we see that  $f$  and  $f^{-1}$  are reflections about the line  $y = x$ .



29. Reflect the graph of  $f$  about the line  $y = x$ . The points  $(-1, -2)$ ,  $(1, -1)$ ,  $(2, 2)$ , and  $(3, 3)$  on  $f$  are reflected to  $(-2, -1)$ ,  $(-1, 1)$ ,  $(2, 2)$ , and  $(3, 3)$  on  $f^{-1}$ .



30. Reflect the graph of  $f$  about the line  $y = x$ .

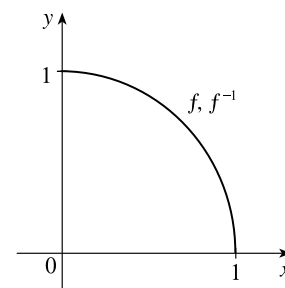


31. (a)  $y = f(x) = \sqrt{1-x^2}$  ( $0 \leq x \leq 1$  and note that  $y \geq 0$ )  $\Rightarrow$

$$y^2 = 1 - x^2 \Rightarrow x^2 = 1 - y^2 \Rightarrow x = \sqrt{1 - y^2}. \text{ So}$$

$f^{-1}(x) = \sqrt{1-x^2}$ ,  $0 \leq x \leq 1$ . We see that  $f^{-1}$  and  $f$  are the same function.

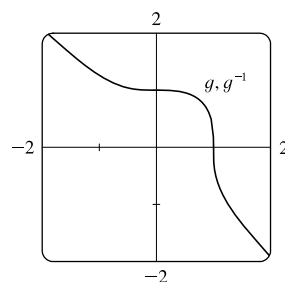
(b) The graph of  $f$  is the portion of the circle  $x^2 + y^2 = 1$  with  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$  (quarter-circle in the first quadrant). The graph of  $f$  is symmetric with respect to the line  $y = x$ , so its reflection about  $y = x$  is itself, that is,  $f^{-1} = f$ .



32. (a)  $y = g(x) = \sqrt[3]{1-x^3} \Rightarrow y^3 = 1 - x^3 \Rightarrow x^3 = 1 - y^3 \Rightarrow$

$x = \sqrt[3]{1-y^3}$ . So  $g^{-1}(x) = \sqrt[3]{1-x^3}$ . We see that  $g$  and  $g^{-1}$  are the same function.

(b) The graph of  $g$  is symmetric with respect to the line  $y = x$ , so its reflection about  $y = x$  is itself, that is,  $g^{-1} = g$ .



33. (a) It is defined as the inverse of the exponential function with base  $b$ , that is,  $\log_b x = y \Leftrightarrow b^y = x$ .

(b)  $(0, \infty)$                       (c)  $\mathbb{R}$                       (d) See Figure 11.

34. (a) The natural logarithm is the logarithm with base  $e$ , denoted  $\ln x$ .

(b) The common logarithm is the logarithm with base 10, denoted  $\log x$ .

(c) See Figure 13.

35. (a)  $\log_2 32 = \log_2 2^5 = 5$  by (7).

(b)  $\log_8 2 = \log_8 8^{1/3} = \frac{1}{3}$  by (7).

*Another method:* Set the logarithm equal to  $x$  and change to an exponential equation.

$$\log_8 2 = x \Leftrightarrow 8^x = 2 \Leftrightarrow (2^3)^x = 2 \Leftrightarrow 2^{3x} = 2^1 \Leftrightarrow 3x = 1 \Leftrightarrow x = \frac{1}{3}.$$

36. (a)  $\log_5 \frac{1}{125} = \log_5 \frac{1}{5^3} = \log_5 5^{-3} = -3$  by (7).

(b)  $\ln(1/e^2) = \ln e^{-2} = -2$  by (9).

37. (a)  $\log_{10} 40 + \log_{10} 2.5 = \log_{10} [(40)(2.5)]$  [by Law 1]

$$= \log_{10} 100$$

$$= \log_{10} 10^2 = 2 \quad \text{[by (7)]}$$

# NOT FOR SALE

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$$\begin{aligned}
 \text{(b) } \log_8 60 - \log_8 3 - \log_8 5 &= \log_8 \frac{60}{3} - \log_8 5 && \text{[by Law 2]} \\
 &= \log_8 20 - \log_8 5 \\
 &= \log_8 \frac{20}{5} && \text{[by Law 2]} \\
 &= \log_8 4 = \log_8 8^{2/3} = \frac{2}{3} && \text{[by (7)]}
 \end{aligned}$$

38. (a)  $e^{-\ln 2} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$  by (9). Or:  $e^{-\ln 2} = (e^{\ln 2})^{-1} = 2^{-1} = \frac{1}{2}$

(b)  $e^{\ln(\ln e^3)} = e^{\ln 3}$  [by (9)] = 3 by (9).

39.  $\ln 10 + 2 \ln 5 = \ln 10 + \ln 5^2$  [by Law 3]  
 $= \ln [(10)(25)]$  [by Law 1]  
 $= \ln 250$

40.  $\ln b + 2 \ln c - 3 \ln d = \ln b + \ln c^2 - \ln d^3$  [by Law 3]  
 $= \ln bc^2 - \ln d^3$  [by Law 1]  
 $= \ln \frac{bc^2}{d^3}$  [by Law 2]

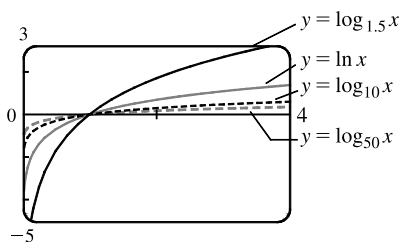
41.  $\frac{1}{3} \ln(x+2)^3 + \frac{1}{2} [\ln x - \ln(x^2 + 3x + 2)^2] = \ln[(x+2)^3]^{1/3} + \frac{1}{2} \ln \frac{x}{(x^2 + 3x + 2)^2}$  [by Laws 3, 2]  
 $= \ln(x+2) + \ln \frac{\sqrt{x}}{x^2 + 3x + 2}$  [by Law 3]  
 $= \ln \frac{(x+2)\sqrt{x}}{(x+1)(x+2)}$  [by Law 1]  
 $= \ln \frac{\sqrt{x}}{x+1}$

Note that since  $\ln x$  is defined for  $x > 0$ , we have  $x + 1$ ,  $x + 2$ , and  $x^2 + 3x + 2$  all positive, and hence their logarithms are defined.

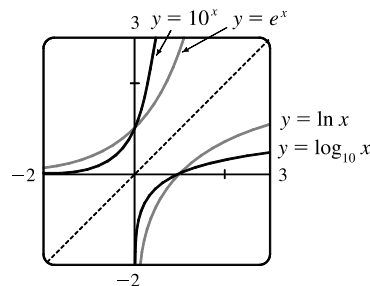
42. (a)  $\log_5 10 = \frac{\ln 10}{\ln 5}$  [by (10)]  $\approx 1.430677$       (b)  $\log_3 57 = \frac{\ln 57}{\ln 3}$  [by (10)]  $\approx 3.680144$

43. To graph these functions, we use  $\log_{1.5} x = \frac{\ln x}{\ln 1.5}$  and  $\log_{50} x = \frac{\ln x}{\ln 50}$ .

These graphs all approach  $-\infty$  as  $x \rightarrow 0^+$ , and they all pass through the point  $(1, 0)$ . Also, they are all increasing, and all approach  $\infty$  as  $x \rightarrow \infty$ . The functions with larger bases increase extremely slowly, and the ones with smaller bases do so somewhat more quickly. The functions with large bases approach the  $y$ -axis more closely as  $x \rightarrow 0^+$ .



44. We see that the graph of  $\ln x$  is the reflection of the graph of  $e^x$  about the line  $y = x$ , and that the graph of  $\log_{10} x$  is the reflection of the graph of  $10^x$  about the same line. The graph of  $10^x$  increases more quickly than that of  $e^x$ . Also note that  $\log_{10} x \rightarrow \infty$  as  $x \rightarrow \infty$  more slowly than  $\ln x$ .

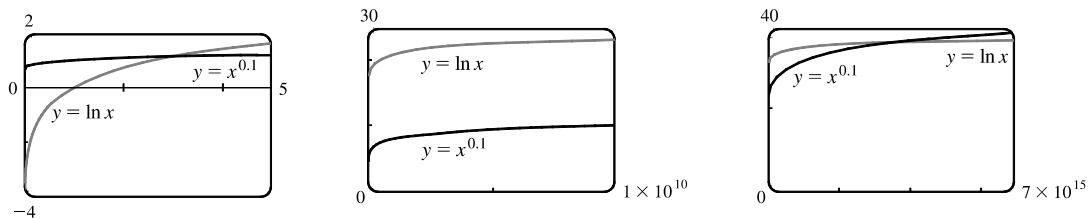




45. 3 ft = 36 in, so we need  $x$  such that  $\log_2 x = 36 \Leftrightarrow x = 2^{36} = 68,719,476,736$ . In miles, this is

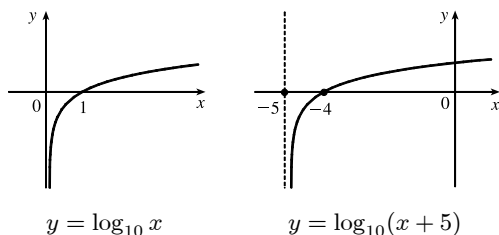
$$68,719,476,736 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 1,084,587.7 \text{ mi.}$$

46.

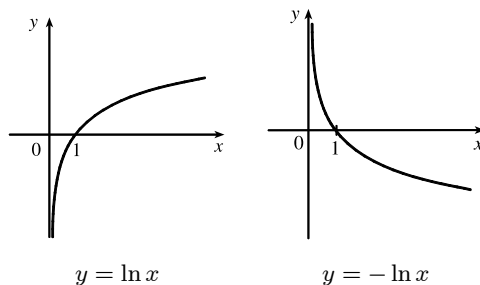


From the graphs, we see that  $f(x) = x^{0.1} > g(x) = \ln x$  for approximately  $0 < x < 3.06$ , and then  $g(x) > f(x)$  for  $3.06 < x < 3.43 \times 10^{15}$  (approximately). At that point, the graph of  $f$  finally surpasses the graph of  $g$  for good.

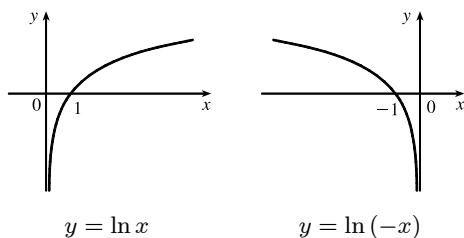
47. (a) Shift the graph of  $y = \log_{10} x$  five units to the left to obtain the graph of  $y = \log_{10}(x + 5)$ . Note the vertical asymptote of  $x = -5$ .



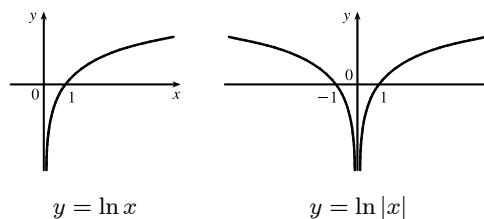
(b) Reflect the graph of  $y = \ln x$  about the  $x$ -axis to obtain the graph of  $y = -\ln x$ .



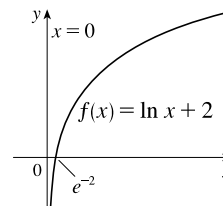
48. (a) Reflect the graph of  $y = \ln x$  about the  $y$ -axis to obtain the graph of  $y = \ln(-x)$ .



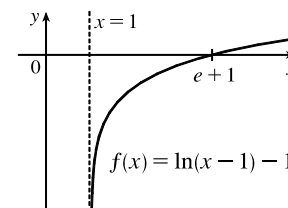
(b) Reflect the portion of the graph of  $y = \ln x$  to the right of the  $y$ -axis about the  $y$ -axis. The graph of  $y = \ln|x|$  is that reflection in addition to the original portion.



49. (a) The domain of  $f(x) = \ln x + 2$  is  $x > 0$  and the range is  $\mathbb{R}$ .  
 (b)  $y = 0 \Rightarrow \ln x + 2 = 0 \Rightarrow \ln x = -2 \Rightarrow x = e^{-2}$   
 (c) We shift the graph of  $y = \ln x$  two units upward.



50. (a) The domain of  $f(x) = \ln(x - 1) - 1$  is  $x > 1$  and the range is  $\mathbb{R}$ .  
 (b)  $y = 0 \Rightarrow \ln(x - 1) - 1 = 0 \Rightarrow \ln(x - 1) = 1 \Rightarrow x - 1 = e^1 \Rightarrow x = e + 1$



(c) We shift the graph of  $y = \ln x$  one unit to the right and one unit downward.

50 □ CHAPTER 1 FUNCTIONS AND MODELS

51. (a)  $e^{7-4x} = 6 \Leftrightarrow 7 - 4x = \ln 6 \Leftrightarrow 7 - \ln 6 = 4x \Leftrightarrow x = \frac{1}{4}(7 - \ln 6)$

(b)  $\ln(3x - 10) = 2 \Leftrightarrow 3x - 10 = e^2 \Leftrightarrow 3x = e^2 + 10 \Leftrightarrow x = \frac{1}{3}(e^2 + 10)$

52. (a)  $\ln(x^2 - 1) = 3 \Leftrightarrow x^2 - 1 = e^3 \Leftrightarrow x^2 = 1 + e^3 \Leftrightarrow x = \pm\sqrt{1 + e^3}$ .

(b)  $e^{2x} - 3e^x + 2 = 0 \Leftrightarrow (e^x - 1)(e^x - 2) = 0 \Leftrightarrow e^x = 1 \text{ or } e^x = 2 \Leftrightarrow x = \ln 1 \text{ or } x = \ln 2, \text{ so } x = 0 \text{ or } \ln 2$ .

53. (a)  $2^{x-5} = 3 \Leftrightarrow \log_2 3 = x - 5 \Leftrightarrow x = 5 + \log_2 3$ .

Or:  $2^{x-5} = 3 \Leftrightarrow \ln(2^{x-5}) = \ln 3 \Leftrightarrow (x-5)\ln 2 = \ln 3 \Leftrightarrow x-5 = \frac{\ln 3}{\ln 2} \Leftrightarrow x = 5 + \frac{\ln 3}{\ln 2}$

(b)  $\ln x + \ln(x-1) = \ln(x(x-1)) = 1 \Leftrightarrow x(x-1) = e^1 \Leftrightarrow x^2 - x - e = 0$ . The quadratic formula (with  $a = 1$ ,  $b = -1$ , and  $c = -e$ ) gives  $x = \frac{1}{2}(1 \pm \sqrt{1 + 4e})$ , but we reject the negative root since the natural logarithm is not defined for  $x < 0$ . So  $x = \frac{1}{2}(1 + \sqrt{1 + 4e})$ .

54. (a)  $\ln(\ln x) = 1 \Leftrightarrow e^{\ln(\ln x)} = e^1 \Leftrightarrow \ln x = e^1 = e \Leftrightarrow e^{\ln x} = e^e \Leftrightarrow x = e^e$

(b)  $e^{ax} = Ce^{bx} \Leftrightarrow \ln e^{ax} = \ln[C(e^{bx})] \Leftrightarrow ax = \ln C + \ln e^{bx} \Leftrightarrow ax = \ln C + bx \Leftrightarrow$

$ax - bx = \ln C \Leftrightarrow (a-b)x = \ln C \Leftrightarrow x = \frac{\ln C}{a-b}$

55. (a)  $\ln x < 0 \Rightarrow x < e^0 \Rightarrow x < 1$ . Since the domain of  $f(x) = \ln x$  is  $x > 0$ , the solution of the original inequality is  $0 < x < 1$ .

(b)  $e^x > 5 \Rightarrow \ln e^x > \ln 5 \Rightarrow x > \ln 5$

56. (a)  $1 < e^{3x-1} < 2 \Rightarrow \ln 1 < 3x-1 < \ln 2 \Rightarrow 0 < 3x-1 < \ln 2 \Rightarrow 1 < 3x < 1 + \ln 2 \Rightarrow \frac{1}{3} < x < \frac{1}{3}(1 + \ln 2)$

(b)  $1 - 2 \ln x < 3 \Rightarrow -2 \ln x < 2 \Rightarrow \ln x > -1 \Rightarrow x > e^{-1}$

57. (a) We must have  $e^x - 3 > 0 \Leftrightarrow e^x > 3 \Leftrightarrow x > \ln 3$ . Thus, the domain of  $f(x) = \ln(e^x - 3)$  is  $(\ln 3, \infty)$ .

(b)  $y = \ln(e^x - 3) \Rightarrow e^y = e^x - 3 \Rightarrow e^x = e^y + 3 \Rightarrow x = \ln(e^y + 3)$ , so  $f^{-1}(x) = \ln(e^x + 3)$ .

Now  $e^x + 3 > 0 \Rightarrow e^x > -3$ , which is true for any real  $x$ , so the domain of  $f^{-1}$  is  $\mathbb{R}$ .

58. (a) By (9),  $e^{\ln 300} = 300$  and  $\ln(e^{300}) = 300$ .

(b) A calculator gives  $e^{\ln 300} = 300$  and an error message for  $\ln(e^{300})$  since  $e^{300}$  is larger than most calculators can evaluate.

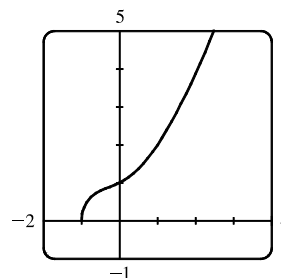
59. We see that the graph of  $y = f(x) = \sqrt{x^3 + x^2 + x + 1}$  is increasing, so  $f$  is 1-1.

Enter  $x = \sqrt{y^3 + y^2 + y + 1}$  and use your CAS to solve the equation for  $y$ .

Using Derive, we get two (irrelevant) solutions involving imaginary expressions, as well as one which can be simplified to the following:

$$y = f^{-1}(x) = -\frac{\sqrt[3]{4}}{6} (\sqrt[3]{D - 27x^2 + 20} - \sqrt[3]{D + 27x^2 - 20} + \sqrt[3]{2})$$

where  $D = 3\sqrt{3}\sqrt{27x^4 - 40x^2 + 16}$ .



[continued]

Maple and Mathematica each give two complex expressions and one real expression, and the real expression is equivalent to that given by Derive. For example, Maple's expression simplifies to  $\frac{1}{6} \frac{M^{2/3} - 8 - 2M^{1/3}}{2M^{1/3}}$ , where  $M = 108x^2 + 12\sqrt{48 - 120x^2 + 81x^4} - 80$ .

60. (a) If we use Derive, then solving  $x = y^6 + y^4$  for  $y$  gives us six solutions of the form  $y = \pm \frac{\sqrt{3}}{3} \sqrt{B-1}$ , where

$$B \in \left\{ -2 \sin \frac{A}{3}, 2 \sin \left( \frac{A}{3} + \frac{\pi}{3} \right), -2 \cos \left( \frac{A}{3} + \frac{\pi}{6} \right) \right\} \text{ and } A = \sin^{-1} \left( \frac{27x-2}{2} \right).$$

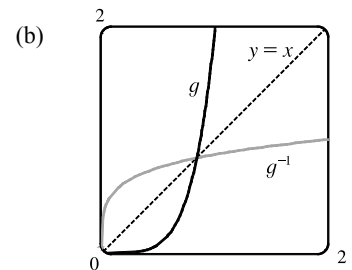
The inverse for  $y = x^6 + x^4$  ( $x \geq 0$ ) is  $y = \frac{\sqrt{3}}{3} \sqrt{B-1}$  with  $B = 2 \sin \left( \frac{A}{3} + \frac{\pi}{3} \right)$ , but because the domain of  $A$  is  $[0, \frac{4}{27}]$ , this expression is only valid for  $x \in [0, \frac{4}{27}]$ .

Happily, Maple gives us the rest of the solution! We solve  $x = y^6 + y^4$  for  $y$  to get the two real solutions  $\pm \frac{\sqrt{6}}{6} \frac{\sqrt{C^{1/3}(C^{2/3} - 2C^{1/3} + 4)}}{C^{1/3}}$ , where  $C = 108x + 12\sqrt{3}\sqrt{x(27x-4)}$ , and the inverse for  $y = x^6 + x^4$  ( $x \geq 0$ ) is the positive solution, whose domain is  $[\frac{4}{27}, \infty)$ .

Mathematica also gives two real solutions, equivalent to those of Maple.

The positive one is  $\frac{\sqrt{6}}{6} \left( \sqrt[3]{4D^{1/3}} + 2\sqrt[3]{2D^{-1/3}} - 2 \right)$ , where

$D = -2 + 27x + 3\sqrt{3}\sqrt{x}\sqrt{27x-4}$ . Although this expression also has domain  $[\frac{4}{27}, \infty)$ , Mathematica is mysteriously able to plot the solution for all  $x \geq 0$ .



61. (a)  $n = f(t) = 100 \cdot 2^{t/3} \Rightarrow \frac{n}{100} = 2^{t/3} \Rightarrow \log_2 \left( \frac{n}{100} \right) = \frac{t}{3} \Rightarrow t = 3 \log_2 \left( \frac{n}{100} \right)$ . Using formula (10), we can write this as  $t = f^{-1}(n) = 3 \cdot \frac{\ln(n/100)}{\ln 2}$ . This function tells us how long it will take to obtain  $n$  bacteria (given the number  $n$ ).

(b)  $n = 50,000 \Rightarrow t = f^{-1}(50,000) = 3 \cdot \frac{\ln \left( \frac{50,000}{100} \right)}{\ln 2} = 3 \left( \frac{\ln 500}{\ln 2} \right) \approx 26.9$  hours

62. (a)  $Q = Q_0(1 - e^{-t/a}) \Rightarrow \frac{Q}{Q_0} = 1 - e^{-t/a} \Rightarrow e^{-t/a} = 1 - \frac{Q}{Q_0} \Rightarrow -\frac{t}{a} = \ln \left( 1 - \frac{Q}{Q_0} \right) \Rightarrow t = -a \ln \left( 1 - \frac{Q}{Q_0} \right)$ . This gives us the time  $t$  necessary to obtain a given charge  $Q$ .

(b)  $Q = 0.9Q_0$  and  $a = 2 \Rightarrow t = -2 \ln \left( 1 - 0.9Q_0/Q_0 \right) = -2 \ln 0.1 \approx 4.6$  seconds.

63. (a)  $\cos^{-1}(-1) = \pi$  because  $\cos \pi = -1$  and  $\pi$  is in the interval  $[0, \pi]$  (the range of  $\cos^{-1}$ ).

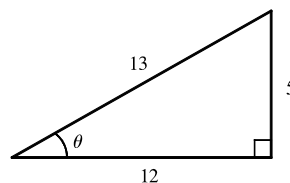
(b)  $\sin^{-1}(0.5) = \frac{\pi}{6}$  because  $\sin \frac{\pi}{6} = 0.5$  and  $\frac{\pi}{6}$  is in the interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  (the range of  $\sin^{-1}$ ).

64. (a)  $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$  because  $\tan \frac{\pi}{3} = \sqrt{3}$  and  $\frac{\pi}{3}$  is in the interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$  (the range of  $\tan^{-1}$ ).

(b)  $\arctan(-1) = -\frac{\pi}{4}$  because  $\tan(-\frac{\pi}{4}) = -1$  and  $-\frac{\pi}{4}$  is in the interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$  (the range of  $\arctan$ ).

65. (a)  $\csc^{-1}\sqrt{2} = \frac{\pi}{4}$  because  $\csc \frac{\pi}{4} = \sqrt{2}$  and  $\frac{\pi}{4}$  is in  $(0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$  (the range of  $\csc^{-1}$ ).  
 (b)  $\arcsin 1 = \frac{\pi}{2}$  because  $\sin \frac{\pi}{2} = 1$  and  $\frac{\pi}{2}$  is in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  (the range of  $\arcsin$ ).
66. (a)  $\sin^{-1}(-1/\sqrt{2}) = -\frac{\pi}{4}$  because  $\sin(-\frac{\pi}{4}) = -1/\sqrt{2}$  and  $-\frac{\pi}{4}$  is in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .  
 (b)  $\cos^{-1}(\sqrt{3}/2) = \frac{\pi}{6}$  because  $\cos \frac{\pi}{6} = \sqrt{3}/2$  and  $\frac{\pi}{6}$  is in  $[0, \pi]$ .
67. (a)  $\cot^{-1}(-\sqrt{3}) = \frac{5\pi}{6}$  because  $\cot \frac{5\pi}{6} = -\sqrt{3}$  and  $\frac{5\pi}{6}$  is in  $(0, \pi)$  (the range of  $\cot^{-1}$ ).  
 (b)  $\sec^{-1} 2 = \frac{\pi}{3}$  because  $\sec \frac{\pi}{3} = 2$  and  $\frac{\pi}{3}$  is in  $[0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2})$  (the range of  $\sec^{-1}$ ).
68. (a)  $\arcsin(\sin(5\pi/4)) = \arcsin(-1/\sqrt{2}) = -\frac{\pi}{4}$  because  $\sin(-\frac{\pi}{4}) = -1/\sqrt{2}$  and  $-\frac{\pi}{4}$  is in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .  
 (b) Let  $\theta = \sin^{-1}(\frac{5}{13})$  [see the figure].

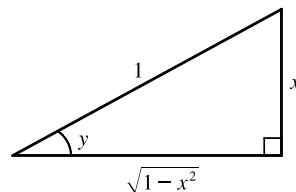
$$\begin{aligned} \cos(2 \sin^{-1}(\frac{5}{13})) &= \cos 2\theta = \cos^2 \theta - \sin^2 \theta \\ &= (\frac{12}{13})^2 - (\frac{5}{13})^2 = \frac{144}{169} - \frac{25}{169} = \frac{119}{169} \end{aligned}$$



69. Let  $y = \sin^{-1} x$ . Then  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \Rightarrow \cos y \geq 0$ , so  $\cos(\sin^{-1} x) = \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$ .

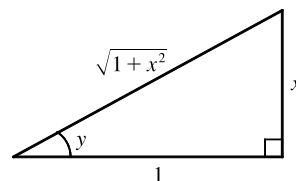
70. Let  $y = \sin^{-1} x$ . Then  $\sin y = x$ , so from the triangle (which illustrates the case  $y > 0$ ), we see that

$$\tan(\sin^{-1} x) = \tan y = \frac{x}{\sqrt{1-x^2}}$$



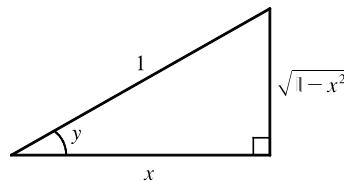
71. Let  $y = \tan^{-1} x$ . Then  $\tan y = x$ , so from the triangle (which illustrates the case  $y > 0$ ), we see that

$$\sin(\tan^{-1} x) = \sin y = \frac{x}{\sqrt{1+x^2}}$$

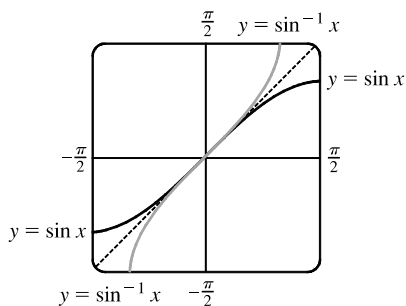


72. Let  $y = \arccos x$ . Then  $\cos y = x$ , so from the triangle (which illustrates the case  $y > 0$ ), we see that

$$\begin{aligned} \sin(2 \arccos x) &= \sin 2y = 2 \sin y \cos y \\ &= 2(\sqrt{1-x^2})(x) = 2x\sqrt{1-x^2} \end{aligned}$$

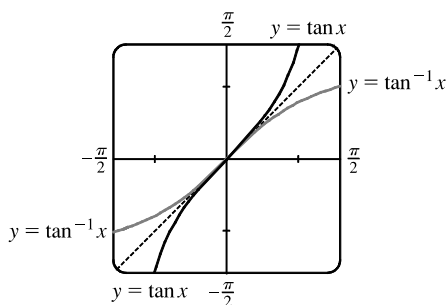


- 73.



The graph of  $\sin^{-1} x$  is the reflection of the graph of  $\sin x$  about the line  $y = x$ .

74.



The graph of  $\tan^{-1} x$  is the reflection of the graph of  $\tan x$  about the line  $y = x$ .

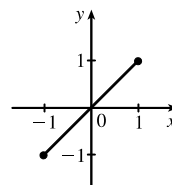
75.  $g(x) = \sin^{-1}(3x + 1)$ .

Domain  $(g) = \{x \mid -1 \leq 3x + 1 \leq 1\} = \{x \mid -2 \leq 3x \leq 0\} = \{x \mid -\frac{2}{3} \leq x \leq 0\} = [-\frac{2}{3}, 0]$ .

Range  $(g) = \{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\} = [-\frac{\pi}{2}, \frac{\pi}{2}]$ .

76. (a)  $f(x) = \sin(\sin^{-1} x)$

Since one function undoes what the other one does, we get the identity function,  $y = x$ , on the restricted domain  $-1 \leq x \leq 1$ .



(b)  $g(x) = \sin^{-1}(\sin x)$

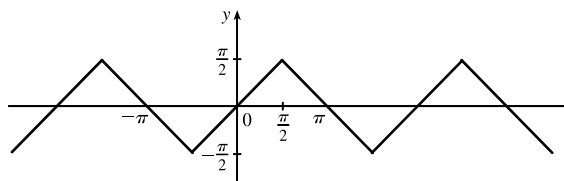
This is similar to part (a), but with domain  $\mathbb{R}$ .

Equations for  $g$  on intervals of the form

$(-\frac{\pi}{2} + \pi n, \frac{\pi}{2} + \pi n)$ , for any integer  $n$ , can be

found using  $g(x) = (-1)^n x + (-1)^{n+1} n\pi$ .

The sine function is monotonic on each of these intervals, and hence, so is  $g$  (but in a linear fashion).



77. (a) If the point  $(x, y)$  is on the graph of  $y = f(x)$ , then the point  $(x - c, y)$  is that point shifted  $c$  units to the left. Since  $f$  is 1-1, the point  $(y, x)$  is on the graph of  $y = f^{-1}(x)$  and the point corresponding to  $(x - c, y)$  on the graph of  $f$  is  $(y, x - c)$  on the graph of  $f^{-1}$ . Thus, the curve's reflection is shifted *down* the same number of units as the curve itself is shifted to the left. So an expression for the inverse function is  $g^{-1}(x) = f^{-1}(x) - c$ .

(b) If we compress (or stretch) a curve horizontally, the curve's reflection in the line  $y = x$  is compressed (or stretched) *vertically* by the same factor. Using this geometric principle, we see that the inverse of  $h(x) = f(cx)$  can be expressed as  $h^{-1}(x) = (1/c) f^{-1}(x)$ .

## 1 Review

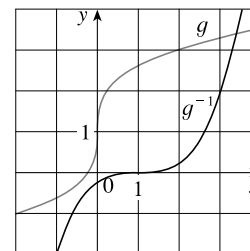
## TRUE-FALSE QUIZ

1. False. Let  $f(x) = x^2$ ,  $s = -1$ , and  $t = 1$ . Then  $f(s + t) = (-1 + 1)^2 = 0^2 = 0$ , but  $f(s) + f(t) = (-1)^2 + 1^2 = 2 \neq 0 = f(s + t)$ .
2. False. Let  $f(x) = x^2$ . Then  $f(-2) = 4 = f(2)$ , but  $-2 \neq 2$ .
3. False. Let  $f(x) = x^2$ . Then  $f(3x) = (3x)^2 = 9x^2$  and  $3f(x) = 3x^2$ . So  $f(3x) \neq 3f(x)$ .
4. True. If  $x_1 < x_2$  and  $f$  is a decreasing function, then the  $y$ -values get smaller as we move from left to right. Thus,  $f(x_1) > f(x_2)$ .
5. True. See the Vertical Line Test.
6. False. Let  $f(x) = x^2$  and  $g(x) = 2x$ . Then  $(f \circ g)(x) = f(g(x)) = f(2x) = (2x)^2 = 4x^2$  and  $(g \circ f)(x) = g(f(x)) = g(x^2) = 2x^2$ . So  $f \circ g \neq g \circ f$ .
7. False. Let  $f(x) = x^3$ . Then  $f$  is one-to-one and  $f^{-1}(x) = \sqrt[3]{x}$ . But  $1/f(x) = 1/x^3$ , which is not equal to  $f^{-1}(x)$ .
8. True. We can divide by  $e^x$  since  $e^x \neq 0$  for every  $x$ .
9. True. The function  $\ln x$  is an increasing function on  $(0, \infty)$ .
10. False. Let  $x = e$ . Then  $(\ln x)^6 = (\ln e)^6 = 1^6 = 1$ , but  $6 \ln x = 6 \ln e = 6 \cdot 1 = 6 \neq 1 = (\ln x)^6$ . What is true, however, is that  $\ln(x^6) = 6 \ln x$  for  $x > 0$ .
11. False. Let  $x = e^2$  and  $a = e$ . Then  $\frac{\ln x}{\ln a} = \frac{\ln e^2}{\ln e} = \frac{2 \ln e}{\ln e} = 2$  and  $\ln \frac{x}{a} = \ln \frac{e^2}{e} = \ln e = 1$ , so in general the statement is false. What is true, however, is that  $\ln \frac{x}{a} = \ln x - \ln a$ .
12. False. It is true that  $\tan \frac{3\pi}{4} = -1$ , but since the range of  $\tan^{-1}$  is  $(-\frac{\pi}{2}, \frac{\pi}{2})$ , we must have  $\tan^{-1}(-1) = -\frac{\pi}{4}$ .
13. False. For example,  $\tan^{-1} 20$  is defined;  $\sin^{-1} 20$  and  $\cos^{-1} 20$  are not.
14. False. For example, if  $x = -3$ , then  $\sqrt{(-3)^2} = \sqrt{9} = 3$ , not  $-3$ .

## EXERCISES

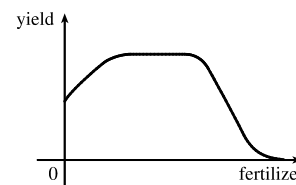
1. (a) When  $x = 2$ ,  $y \approx 2.7$ . Thus,  $f(2) \approx 2.7$ .  
 (b)  $f(x) = 3 \Rightarrow x \approx 2.3, 5.6$   
 (c) The domain of  $f$  is  $-6 \leq x \leq 6$ , or  $[-6, 6]$ .  
 (d) The range of  $f$  is  $-4 \leq y \leq 4$ , or  $[-4, 4]$ .  
 (e)  $f$  is increasing on  $[-4, 4]$ , that is, on  $-4 \leq x \leq 4$ .  
 (f)  $f$  is not one-to-one since it fails the Horizontal Line Test.  
 (g)  $f$  is odd since its graph is symmetric about the origin.

2. (a) When  $x = 2, y = 3$ . Thus,  $g(2) = 3$ .
- (b)  $g$  is one-to-one because it passes the Horizontal Line Test.
- (c) When  $y = 2, x \approx 0.2$ . So  $g^{-1}(2) \approx 0.2$ .
- (d) The range of  $g$  is  $[-1, 3.5]$ , which is the same as the domain of  $g^{-1}$ .
- (e) We reflect the graph of  $g$  through the line  $y = x$  to obtain the graph of  $g^{-1}$ .



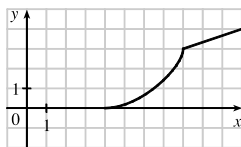
3.  $f(x) = x^2 - 2x + 3$ , so  $f(a + h) = (a + h)^2 - 2(a + h) + 3 = a^2 + 2ah + h^2 - 2a - 2h + 3$ , and
 
$$\frac{f(a + h) - f(a)}{h} = \frac{(a^2 + 2ah + h^2 - 2a - 2h + 3) - (a^2 - 2a + 3)}{h} = \frac{h(2a + h - 2)}{h} = 2a + h - 2.$$

4. There will be some yield with no fertilizer, increasing yields with increasing fertilizer use, a leveling-off of yields at some point, and disaster with too much fertilizer use.

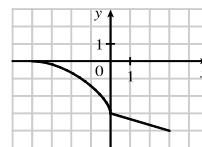


5.  $f(x) = 2/(3x - 1)$ . Domain:  $3x - 1 \neq 0 \Rightarrow 3x \neq 1 \Rightarrow x \neq \frac{1}{3}$ .  $D = (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$   
 Range: all reals except 0 ( $y = 0$  is the horizontal asymptote for  $f$ ).  
 $R = (-\infty, 0) \cup (0, \infty)$
6.  $g(x) = \sqrt{16 - x^4}$ . Domain:  $16 - x^4 \geq 0 \Rightarrow x^4 \leq 16 \Rightarrow |x| \leq \sqrt[4]{16} \Rightarrow |x| \leq 2$ .  $D = [-2, 2]$   
 Range:  $y \geq 0$  and  $y \leq \sqrt{16} \Rightarrow 0 \leq y \leq 4$ .  
 $R = [0, 4]$
7.  $h(x) = \ln(x + 6)$ . Domain:  $x + 6 > 0 \Rightarrow x > -6$ .  $D = (-6, \infty)$   
 Range:  $x + 6 > 0$ , so  $\ln(x + 6)$  takes on all real numbers and, hence, the range is  $\mathbb{R}$ .  
 $R = (-\infty, \infty)$
8.  $y = F(t) = 3 + \cos 2t$ . Domain:  $\mathbb{R}$ .  $D = (-\infty, \infty)$   
 Range:  $-1 \leq \cos 2t \leq 1 \Rightarrow 2 \leq 3 + \cos 2t \leq 4 \Rightarrow 2 \leq y \leq 4$ .  
 $R = [2, 4]$
9. (a) To obtain the graph of  $y = f(x) + 8$ , we shift the graph of  $y = f(x)$  up 8 units.
- (b) To obtain the graph of  $y = f(x + 8)$ , we shift the graph of  $y = f(x)$  left 8 units.
- (c) To obtain the graph of  $y = 1 + 2f(x)$ , we stretch the graph of  $y = f(x)$  vertically by a factor of 2, and then shift the resulting graph 1 unit upward.
- (d) To obtain the graph of  $y = f(x - 2) - 2$ , we shift the graph of  $y = f(x)$  right 2 units (for the “-2” inside the parentheses), and then shift the resulting graph 2 units downward.
- (e) To obtain the graph of  $y = -f(x)$ , we reflect the graph of  $y = f(x)$  about the  $x$ -axis.
- (f) To obtain the graph of  $y = f^{-1}(x)$ , we reflect the graph of  $y = f(x)$  about the line  $y = x$  (assuming  $f$  is one-to-one).

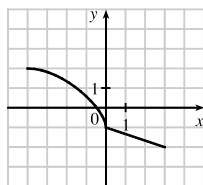
10. (a) To obtain the graph of  $y = f(x - 8)$ , we shift the graph of  $y = f(x)$  right 8 units.



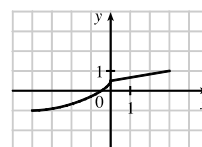
- (b) To obtain the graph of  $y = -f(x)$ , we reflect the graph of  $y = f(x)$  about the  $x$ -axis.



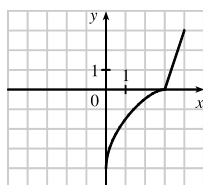
- (c) To obtain the graph of  $y = 2 - f(x)$ , we reflect the graph of  $y = f(x)$  about the  $x$ -axis, and then shift the resulting graph 2 units upward.



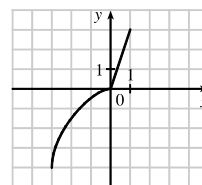
- (d) To obtain the graph of  $y = \frac{1}{2}f(x) - 1$ , we shrink the graph of  $y = f(x)$  by a factor of 2, and then shift the resulting graph 1 unit downward.



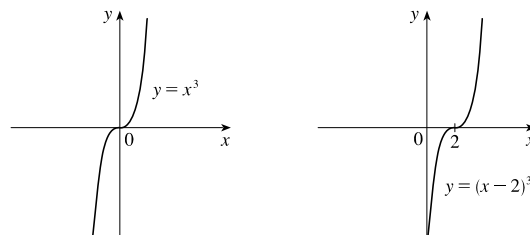
- (e) To obtain the graph of  $y = f^{-1}(x)$ , we reflect the graph of  $y = f(x)$  about the line  $y = x$ .



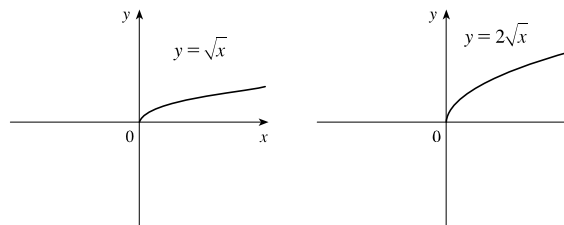
- (f) To obtain the graph of  $y = f^{-1}(x + 3)$ , we reflect the graph of  $y = f(x)$  about the line  $y = x$  [see part (e)], and then shift the resulting graph left 3 units.



11.  $y = (x - 2)^3$ : Start with the graph of  $y = x^3$  and shift 2 units to the right.

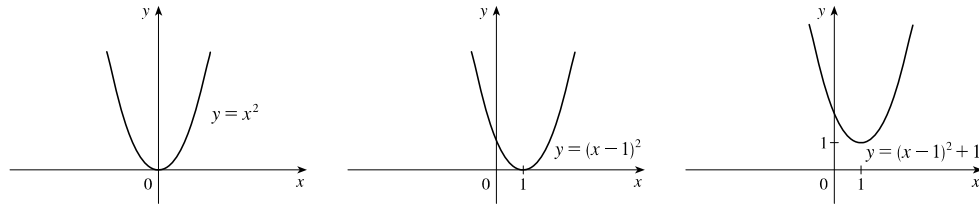


12.  $y = 2\sqrt{x}$ : Start with the graph of  $y = \sqrt{x}$  and stretch vertically by a factor of 2.

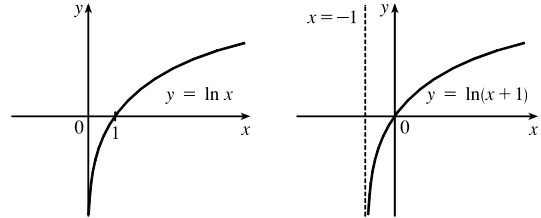




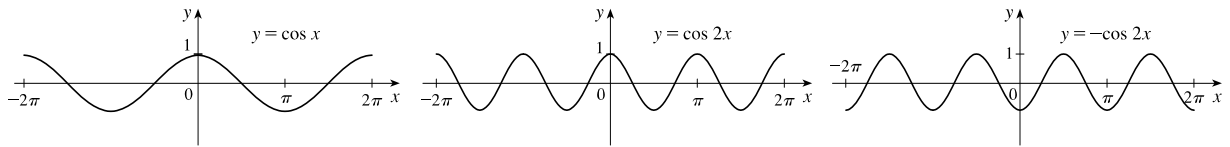
13.  $y = x^2 - 2x + 2 = (x^2 - 2x + 1) + 1 = (x - 1)^2 + 1$ : Start with the graph of  $y = x^2$ , shift 1 unit to the right, and shift 1 unit upward.



14.  $y = \ln(x + 1)$ : Start with the graph of  $y = \ln x$  and shift left 1 unit.



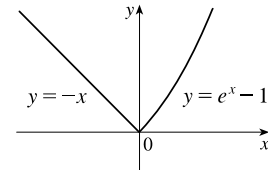
15.  $f(x) = -\cos 2x$ : Start with the graph of  $y = \cos x$ , shrink horizontally by a factor of 2, and reflect about the  $x$ -axis.



16.  $f(x) = \begin{cases} -x & \text{if } x < 0 \\ e^x - 1 & \text{if } x \geq 0 \end{cases}$

On  $(-\infty, 0)$ , graph  $y = -x$  (the line with slope  $-1$  and  $y$ -intercept  $0$ ) with open endpoint  $(0, 0)$ .

On  $[0, \infty)$ , graph  $y = e^x - 1$  (the graph of  $y = e^x$  shifted 1 unit downward) with closed endpoint  $(0, 0)$ .



17. (a) The terms of  $f$  are a mixture of odd and even powers of  $x$ , so  $f$  is neither even nor odd.

(b) The terms of  $f$  are all odd powers of  $x$ , so  $f$  is odd.

(c)  $f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$ , so  $f$  is even.

(d)  $f(-x) = 1 + \sin(-x) = 1 - \sin x$ . Now  $f(-x) \neq f(x)$  and  $f(-x) \neq -f(x)$ , so  $f$  is neither even nor odd.

18. For the line segment from  $(-2, 2)$  to  $(-1, 0)$ , the slope is  $\frac{0 - 2}{-1 + 2} = -2$ , and an equation is  $y - 0 = -2(x + 1)$  or,

equivalently,  $y = -2x - 2$ . The circle has equation  $x^2 + y^2 = 1$ ; the top half has equation  $y = \sqrt{1 - x^2}$  (we have solved for

positive  $y$ ). Thus,  $f(x) = \begin{cases} -2x - 2 & \text{if } -2 \leq x \leq -1 \\ \sqrt{1 - x^2} & \text{if } -1 < x \leq 1 \end{cases}$

19.  $f(x) = \ln x$ ,  $D = (0, \infty)$ ;  $g(x) = x^2 - 9$ ,  $D = \mathbb{R}$ .

(a)  $(f \circ g)(x) = f(g(x)) = f(x^2 - 9) = \ln(x^2 - 9)$ .

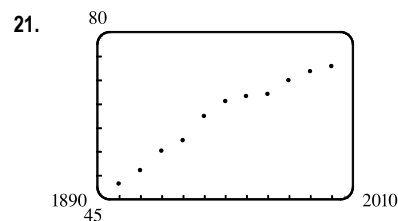
Domain:  $x^2 - 9 > 0 \Rightarrow x^2 > 9 \Rightarrow |x| > 3 \Rightarrow x \in (-\infty, -3) \cup (3, \infty)$

(b)  $(g \circ f)(x) = g(f(x)) = g(\ln x) = (\ln x)^2 - 9$ . Domain:  $x > 0$ , or  $(0, \infty)$

(c)  $(f \circ f)(x) = f(f(x)) = f(\ln x) = \ln(\ln x)$ . Domain:  $\ln x > 0 \Rightarrow x > e^0 = 1$ , or  $(1, \infty)$

(d)  $(g \circ g)(x) = g(g(x)) = g(x^2 - 9) = (x^2 - 9)^2 - 9$ . Domain:  $x \in \mathbb{R}$ , or  $(-\infty, \infty)$

20. Let  $h(x) = x + \sqrt{x}$ ,  $g(x) = \sqrt{x}$ , and  $f(x) = 1/x$ . Then  $(f \circ g \circ h)(x) = \frac{1}{\sqrt{x + \sqrt{x}}} = F(x)$ .

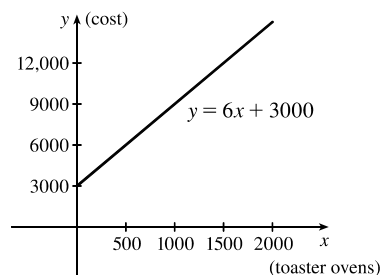


Many models appear to be plausible. Your choice depends on whether you think medical advances will keep increasing life expectancy, or if there is bound to be a natural leveling-off of life expectancy. A linear model,  $y = 0.2493x - 423.4818$ , gives us an estimate of 77.6 years for the year 2010.

22. (a) Let  $x$  denote the number of toaster ovens produced in one week and  $y$  the associated cost. Using the points  $(1000, 9000)$  and  $(1500, 12,000)$ , we get an equation of a line:

$$y - 9000 = \frac{12,000 - 9000}{1500 - 1000}(x - 1000) \Rightarrow$$

$$y = 6(x - 1000) + 9000 \Rightarrow y = 6x + 3000.$$



(b) The slope of 6 means that each additional toaster oven produced adds \$6 to the weekly production cost.

(c) The  $y$ -intercept of 3000 represents the overhead cost—the cost incurred without producing anything.

23. We need to know the value of  $x$  such that  $f(x) = 2x + \ln x = 2$ . Since  $x = 1$  gives us  $y = 2$ ,  $f^{-1}(2) = 1$ .

24.  $y = \frac{x+1}{2x+1}$ . Interchanging  $x$  and  $y$  gives us  $x = \frac{y+1}{2y+1} \Rightarrow 2xy + x = y + 1 \Rightarrow 2xy - y = 1 - x \Rightarrow$

$$y(2x - 1) = 1 - x \Rightarrow y = \frac{1-x}{2x-1} = f^{-1}(x).$$

25. (a)  $e^{2 \ln 3} = (e^{\ln 3})^2 = 3^2 = 9$

(b)  $\log_{10} 25 + \log_{10} 4 = \log_{10}(25 \cdot 4) = \log_{10} 100 = \log_{10} 10^2 = 2$

(c)  $\tan(\arcsin \frac{1}{2}) = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

(d) Let  $\theta = \cos^{-1} \frac{4}{5}$ , so  $\cos \theta = \frac{4}{5}$ . Then  $\sin(\cos^{-1}(\frac{4}{5})) = \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - (\frac{4}{5})^2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$ .

26. (a)  $e^x = 5 \Rightarrow x = \ln 5$

(b)  $\ln x = 2 \Rightarrow x = e^2$

(c)  $e^{e^x} = 2 \Rightarrow e^x = \ln 2 \Rightarrow x = \ln(\ln 2)$

(d)  $\tan^{-1} x = 1 \Rightarrow \tan \tan^{-1} x = \tan 1 \Rightarrow x = \tan 1 (\approx 1.5574)$

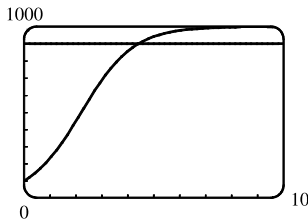
27. (a) After 4 days,  $\frac{1}{2}$  gram remains; after 8 days,  $\frac{1}{4}$  g; after 12 days,  $\frac{1}{8}$  g; after 16 days,  $\frac{1}{16}$  g.

(b)  $m(4) = \frac{1}{2}$ ,  $m(8) = \frac{1}{2^2}$ ,  $m(12) = \frac{1}{2^3}$ ,  $m(16) = \frac{1}{2^4}$ . From the pattern, we see that  $m(t) = \frac{1}{2^{t/4}}$ , or  $2^{-t/4}$ .

(c)  $m = 2^{-t/4} \Rightarrow \log_2 m = -t/4 \Rightarrow t = -4 \log_2 m$ ; this is the time elapsed when there are  $m$  grams of  $^{100}\text{Pd}$ .

(d)  $m = 0.01 \Rightarrow t = -4 \log_2 0.01 = -4 \left( \frac{\ln 0.01}{\ln 2} \right) \approx 26.6$  days

28. (a) The population would reach 900 in about 4.4 years.



(b)  $P = \frac{100,000}{100 + 900e^{-t}} \Rightarrow 100P + 900Pe^{-t} = 100,000 \Rightarrow 900Pe^{-t} = 100,000 - 100P \Rightarrow$

$e^{-t} = \frac{100,000 - 100P}{900P} \Rightarrow -t = \ln\left(\frac{1000 - P}{9P}\right) \Rightarrow t = -\ln\left(\frac{1000 - P}{9P}\right)$ , or  $\ln\left(\frac{9P}{1000 - P}\right)$ ;

this is the time required for the population to reach a given number  $P$ .

(c)  $P = 900 \Rightarrow t = \ln\left(\frac{9 \cdot 900}{1000 - 900}\right) = \ln 81 \approx 4.4$  years, as in part (a).

# NOT FOR SALE

60 □ CHAPTER 1 FUNCTIONS AND MODELS

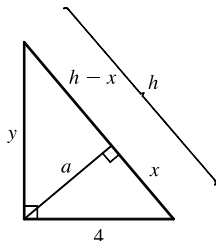
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## □ PRINCIPLES OF PROBLEM SOLVING

1.

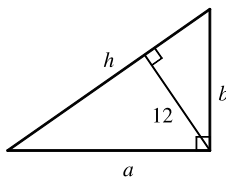


By using the area formula for a triangle,  $\frac{1}{2}$  (base) (height), in two ways, we see that

$$\frac{1}{2}(4)(y) = \frac{1}{2}(h)(a), \text{ so } a = \frac{4y}{h}. \text{ Since } 4^2 + y^2 = h^2, y = \sqrt{h^2 - 16}, \text{ and}$$

$$a = \frac{4\sqrt{h^2 - 16}}{h}.$$

2.



Refer to Example 1, where we obtained  $h = \frac{P^2 - 100}{2P}$ . The 100 came from

4 times the area of the triangle. In this case, the area of the triangle is

$$\frac{1}{2}(h)(12) = 6h. \text{ Thus, } h = \frac{P^2 - 4(6h)}{2P} \Rightarrow 2Ph = P^2 - 24h \Rightarrow$$

$$2Ph + 24h = P^2 \Rightarrow h(2P + 24) = P^2 \Rightarrow h = \frac{P^2}{2P + 24}.$$

$$3. |2x - 1| = \begin{cases} 2x - 1 & \text{if } x \geq \frac{1}{2} \\ 1 - 2x & \text{if } x < \frac{1}{2} \end{cases} \quad \text{and} \quad |x + 5| = \begin{cases} x + 5 & \text{if } x \geq -5 \\ -x - 5 & \text{if } x < -5 \end{cases}$$

Therefore, we consider the three cases  $x < -5$ ,  $-5 \leq x < \frac{1}{2}$ , and  $x \geq \frac{1}{2}$ .

If  $x < -5$ , we must have  $1 - 2x - (-x - 5) = 3 \Leftrightarrow x = 3$ , which is false, since we are considering  $x < -5$ .

If  $-5 \leq x < \frac{1}{2}$ , we must have  $1 - 2x - (x + 5) = 3 \Leftrightarrow x = -\frac{7}{3}$ .

If  $x \geq \frac{1}{2}$ , we must have  $2x - 1 - (x + 5) = 3 \Leftrightarrow x = 9$ .

So the two solutions of the equation are  $x = -\frac{7}{3}$  and  $x = 9$ .

$$4. |x - 1| = \begin{cases} x - 1 & \text{if } x \geq 1 \\ 1 - x & \text{if } x < 1 \end{cases} \quad \text{and} \quad |x - 3| = \begin{cases} x - 3 & \text{if } x \geq 3 \\ 3 - x & \text{if } x < 3 \end{cases}$$

Therefore, we consider the three cases  $x < 1$ ,  $1 \leq x < 3$ , and  $x \geq 3$ .

If  $x < 1$ , we must have  $1 - x - (3 - x) \geq 5 \Leftrightarrow 0 \geq 7$ , which is false.

If  $1 \leq x < 3$ , we must have  $x - 1 - (3 - x) \geq 5 \Leftrightarrow x \geq \frac{9}{2}$ , which is false because  $x < 3$ .

If  $x \geq 3$ , we must have  $x - 1 - (x - 3) \geq 5 \Leftrightarrow 2 \geq 5$ , which is false.

All three cases lead to falsehoods, so the inequality has no solution.

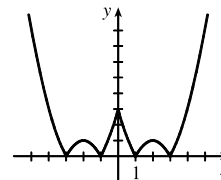
$$5. f(x) = |x^2 - 4|x| + 3|. \text{ If } x \geq 0, \text{ then } f(x) = |x^2 - 4x + 3| = |(x - 1)(x - 3)|.$$

Case (i): If  $0 < x \leq 1$ , then  $f(x) = x^2 - 4x + 3$ .

Case (ii): If  $1 < x \leq 3$ , then  $f(x) = -(x^2 - 4x + 3) = -x^2 + 4x - 3$ .

Case (iii): If  $x > 3$ , then  $f(x) = x^2 - 4x + 3$ .

This enables us to sketch the graph for  $x \geq 0$ . Then we use the fact that  $f$  is an even function to reflect this part of the graph about the  $y$ -axis to obtain the entire graph. Or, we could consider also the cases  $x < -3$ ,  $-3 \leq x < -1$ , and  $-1 \leq x < 0$ .



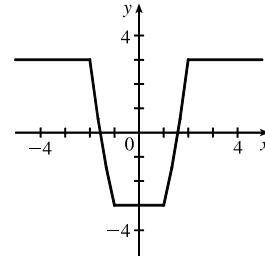
6.  $g(x) = |x^2 - 1| - |x^2 - 4|$ .

$$|x^2 - 1| = \begin{cases} x^2 - 1 & \text{if } |x| \geq 1 \\ 1 - x^2 & \text{if } |x| < 1 \end{cases} \quad \text{and} \quad |x^2 - 4| = \begin{cases} x^2 - 4 & \text{if } |x| \geq 2 \\ 4 - x^2 & \text{if } |x| < 2 \end{cases}$$

So for  $0 \leq |x| < 1$ ,  $g(x) = 1 - x^2 - (4 - x^2) = -3$ , for

$1 \leq |x| < 2$ ,  $g(x) = x^2 - 1 - (4 - x^2) = 2x^2 - 5$ , and for

$|x| \geq 2$ ,  $g(x) = x^2 - 1 - (x^2 - 4) = 3$ .



7. Remember that  $|a| = a$  if  $a \geq 0$  and that  $|a| = -a$  if  $a < 0$ . Thus,

$$x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad \text{and} \quad y + |y| = \begin{cases} 2y & \text{if } y \geq 0 \\ 0 & \text{if } y < 0 \end{cases}$$

We will consider the equation  $x + |x| = y + |y|$  in four cases.

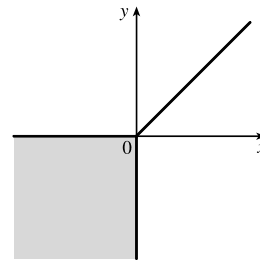
(1) $x \geq 0, y \geq 0$	(2) $x \geq 0, y < 0$	(3) $x < 0, y \geq 0$	(4) $x < 0, y < 0$
$2x = 2y$	$2x = 0$	$0 = 2y$	$0 = 0$
$x = y$	$x = 0$	$0 = y$	

Case 1 gives us the line  $y = x$  with nonnegative  $x$  and  $y$ .

Case 2 gives us the portion of the  $y$ -axis with  $y$  negative.

Case 3 gives us the portion of the  $x$ -axis with  $x$  negative.

Case 4 gives us the entire third quadrant.



8.  $|x - y| + |x| - |y| \leq 2$  [call this inequality (\*)]

Case (i):  $x \geq y \geq 0$ . Then (\*)  $\Leftrightarrow x - y + x - y \leq 2 \Leftrightarrow x - y \leq 1 \Leftrightarrow y \geq x - 1$ .

Case (ii):  $y \geq x \geq 0$ . Then (\*)  $\Leftrightarrow y - x + x - y \leq 2 \Leftrightarrow 0 \leq 2$  (true).

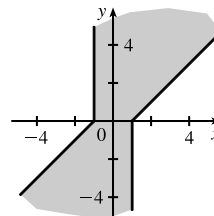
Case (iii):  $x \geq 0$  and  $y \leq 0$ . Then (\*)  $\Leftrightarrow x - y + x + y \leq 2 \Leftrightarrow 2x \leq 2 \Leftrightarrow x \leq 1$ .

Case (iv):  $x \leq 0$  and  $y \geq 0$ . Then (\*)  $\Leftrightarrow y - x - x - y \leq 2 \Leftrightarrow -2x \leq 2 \Leftrightarrow x \geq -1$ .

Case (v):  $y \leq x \leq 0$ . Then (\*)  $\Leftrightarrow x - y - x + y \leq 2 \Leftrightarrow 0 \leq 2$  (true).

Case (vi):  $x \leq y \leq 0$ . Then (\*)  $\Leftrightarrow y - x - x + y \leq 2 \Leftrightarrow y - x \leq 1 \Leftrightarrow y \leq x + 1$ .

Note: Instead of considering cases (iv), (v), and (vi), we could have noted that the region is unchanged if  $x$  and  $y$  are replaced by  $-x$  and  $-y$ , so the region is symmetric about the origin. Therefore, we need only draw cases (i), (ii), and (iii), and rotate through  $180^\circ$  about the origin.



9. (a) To sketch the graph of

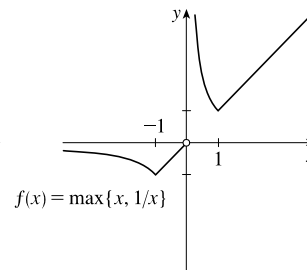
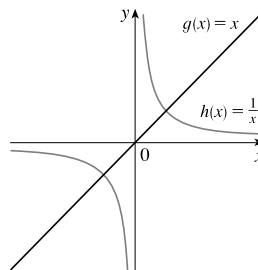
$f(x) = \max\{x, 1/x\}$ , we first graph

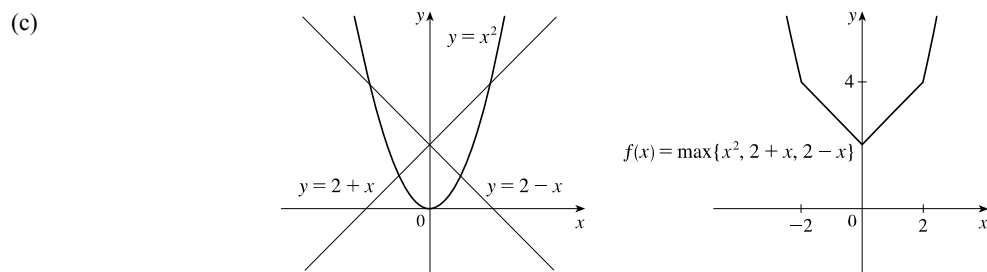
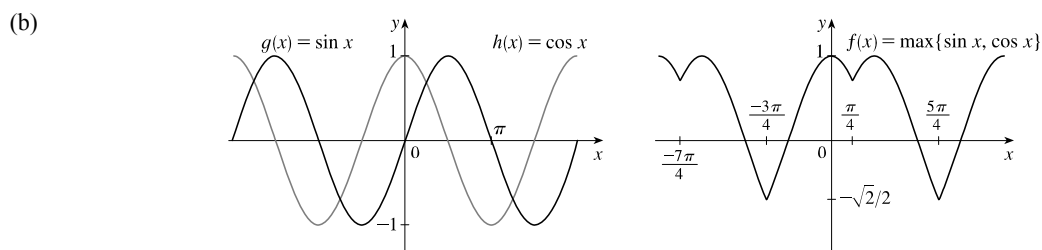
$g(x) = x$  and  $h(x) = 1/x$  on the same

coordinate axes. Then create the graph of

$f$  by plotting the largest  $y$ -value of  $g$  and  $h$

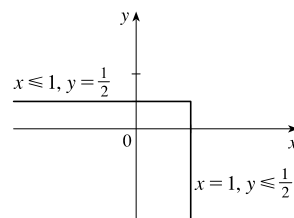
for every value of  $x$ .



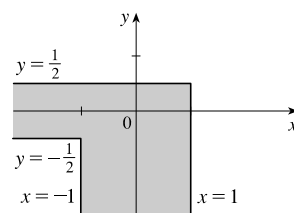


On the TI-84 Plus, max is found under LIST, then under MATH. To graph  $f(x) = \max\{x^2, 2 + x, 2 - x\}$ , use  $Y = \max(x^2, \max(2 + x, 2 - x))$ .

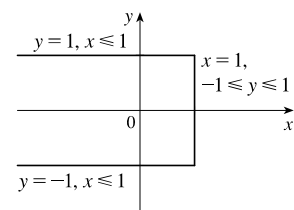
10. (a) If  $\max\{x, 2y\} = 1$ , then either  $x = 1$  and  $2y \leq 1$  or  $x \leq 1$  and  $2y = 1$ . Thus, we obtain the set of points such that  $x = 1$  and  $y \leq \frac{1}{2}$  [a vertical line with highest point  $(1, \frac{1}{2})$ ] or  $x \leq 1$  and  $y = \frac{1}{2}$  [a horizontal line with rightmost point  $(1, \frac{1}{2})$ ].



- (b) The graph of  $\max\{x, 2y\} = 1$  is shown in part (a), and the graph of  $\max\{x, 2y\} = -1$  can be found in a similar manner. The inequalities in  $-1 \leq \max\{x, 2y\} \leq 1$  give us all the points on or inside the boundaries.



- (c)  $\max\{x, y^2\} = 1 \Leftrightarrow$   
 $x = 1$  and  $y^2 \leq 1$  [ $-1 \leq y \leq 1$ ]  
 or  $x \leq 1$  and  $y^2 = 1$  [ $y = \pm 1$ ].



11.  $(\log_2 3)(\log_3 4)(\log_4 5) \cdots (\log_{31} 32) = \left(\frac{\ln 3}{\ln 2}\right) \left(\frac{\ln 4}{\ln 3}\right) \left(\frac{\ln 5}{\ln 4}\right) \cdots \left(\frac{\ln 32}{\ln 31}\right) = \frac{\ln 32}{\ln 2} = \frac{\ln 2^5}{\ln 2} = \frac{5 \ln 2}{\ln 2} = 5$

$$\begin{aligned}
 12. \text{ (a) } f(-x) &= \ln\left(-x + \sqrt{(-x)^2 + 1}\right) = \ln\left(-x + \sqrt{x^2 + 1} \cdot \frac{-x - \sqrt{x^2 + 1}}{-x - \sqrt{x^2 + 1}}\right) \\
 &= \ln\left(\frac{x^2 - (x^2 + 1)}{-x - \sqrt{x^2 + 1}}\right) = \ln\left(\frac{-1}{-x - \sqrt{x^2 + 1}}\right) = \ln\left(\frac{1}{x + \sqrt{x^2 + 1}}\right) \\
 &= \ln 1 - \ln(x + \sqrt{x^2 + 1}) = -\ln(x + \sqrt{x^2 + 1}) = -f(x)
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } y &= \ln(x + \sqrt{x^2 + 1}). \text{ Interchanging } x \text{ and } y, \text{ we get } x = \ln(y + \sqrt{y^2 + 1}) \Rightarrow e^x = y + \sqrt{y^2 + 1} \Rightarrow \\
 e^x - y &= \sqrt{y^2 + 1} \Rightarrow e^{2x} - 2ye^x + y^2 = y^2 + 1 \Rightarrow e^{2x} - 1 = 2ye^x \Rightarrow y = \frac{e^{2x} - 1}{2e^x} = f^{-1}(x)
 \end{aligned}$$

$$13. \ln(x^2 - 2x - 2) \leq 0 \Rightarrow x^2 - 2x - 2 \leq e^0 = 1 \Rightarrow x^2 - 2x - 3 \leq 0 \Rightarrow (x - 3)(x + 1) \leq 0 \Rightarrow x \in [-1, 3].$$

$$\text{Since the argument must be positive, } x^2 - 2x - 2 > 0 \Rightarrow [x - (1 - \sqrt{3})][x - (1 + \sqrt{3})] > 0 \Rightarrow$$

$$x \in (-\infty, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, \infty). \text{ The intersection of these intervals is } [-1, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, 3].$$

14. Assume that  $\log_2 5$  is rational. Then  $\log_2 5 = m/n$  for natural numbers  $m$  and  $n$ . Changing to exponential form gives us  $2^{m/n} = 5$  and then raising both sides to the  $n$ th power gives  $2^m = 5^n$ . But  $2^m$  is even and  $5^n$  is odd. We have arrived at a contradiction, so we conclude that our hypothesis, that  $\log_2 5$  is rational, is false. Thus,  $\log_2 5$  is irrational.

15. Let  $d$  be the distance traveled on each half of the trip. Let  $t_1$  and  $t_2$  be the times taken for the first and second halves of the trip.

For the first half of the trip we have  $t_1 = d/30$  and for the second half we have  $t_2 = d/60$ . Thus, the average speed for the

$$\text{entire trip is } \frac{\text{total distance}}{\text{total time}} = \frac{2d}{t_1 + t_2} = \frac{2d}{\frac{d}{30} + \frac{d}{60}} = \frac{2d}{\frac{2d}{60}} = \frac{120d}{2d + d} = \frac{120d}{3d} = 40. \text{ The average speed for the entire trip}$$

is 40 mi/h.

16. Let  $f(x) = \sin x$ ,  $g(x) = x$ , and  $h(x) = x$ . Then the left-hand side of the equation is

$$[f \circ (g + h)](x) = \sin(x + x) = \sin 2x = 2 \sin x \cos x; \text{ and the right-hand side is}$$

$$(f \circ g)(x) + (f \circ h)(x) = \sin x + \sin x = 2 \sin x. \text{ The two sides are not equal, so the given statement is false.}$$

17. Let  $S_n$  be the statement that  $7^n - 1$  is divisible by 6.

- $S_1$  is true because  $7^1 - 1 = 6$  is divisible by 6.
- Assume  $S_k$  is true, that is,  $7^k - 1$  is divisible by 6. In other words,  $7^k - 1 = 6m$  for some positive integer  $m$ . Then  $7^{k+1} - 1 = 7^k \cdot 7 - 1 = (6m + 1) \cdot 7 - 1 = 42m + 6 = 6(7m + 1)$ , which is divisible by 6, so  $S_{k+1}$  is true.
- Therefore, by mathematical induction,  $7^n - 1$  is divisible by 6 for every positive integer  $n$ .

18. Let  $S_n$  be the statement that  $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ .

- $S_1$  is true because  $[2(1) - 1] = 1 = 1^2$ .
- Assume  $S_k$  is true, that is,  $1 + 3 + 5 + \cdots + (2k - 1) = k^2$ . Then  $1 + 3 + 5 + \cdots + (2k - 1) + [2(k + 1) - 1] = 1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = k^2 + (2k + 1) = (k + 1)^2$  which shows that  $S_{k+1}$  is true.
- Therefore, by mathematical induction,  $1 + 3 + 5 + \cdots + (2n - 1) = n^2$  for every positive integer  $n$ .



19.  $f_0(x) = x^2$  and  $f_{n+1}(x) = f_0(f_n(x))$  for  $n = 0, 1, 2, \dots$

$$f_1(x) = f_0(f_0(x)) = f_0(x^2) = (x^2)^2 = x^4, f_2(x) = f_0(f_1(x)) = f_0(x^4) = (x^4)^2 = x^8,$$

$$f_3(x) = f_0(f_2(x)) = f_0(x^8) = (x^8)^2 = x^{16}, \dots \text{ Thus, a general formula is } f_n(x) = x^{2^{n+1}}.$$

20. (a)  $f_0(x) = 1/(2-x)$  and  $f_{n+1} = f_0 \circ f_n$  for  $n = 0, 1, 2, \dots$

$$f_1(x) = f_0\left(\frac{1}{2-x}\right) = \frac{1}{2 - \frac{1}{2-x}} = \frac{2-x}{2(2-x)-1} = \frac{2-x}{3-2x},$$

$$f_2(x) = f_0\left(\frac{2-x}{3-2x}\right) = \frac{1}{2 - \frac{2-x}{3-2x}} = \frac{3-2x}{2(3-2x)-(2-x)} = \frac{3-2x}{4-3x},$$

$$f_3(x) = f_0\left(\frac{3-2x}{4-3x}\right) = \frac{1}{2 - \frac{3-2x}{4-3x}} = \frac{4-3x}{2(4-3x)-(3-2x)} = \frac{4-3x}{5-4x}, \dots$$

$$\text{Thus, we conjecture that the general formula is } f_n(x) = \frac{n+1-nx}{n+2-(n+1)x}.$$

To prove this, we use the Principle of Mathematical Induction. We have already verified that  $f_n$  is true for  $n = 1$ .

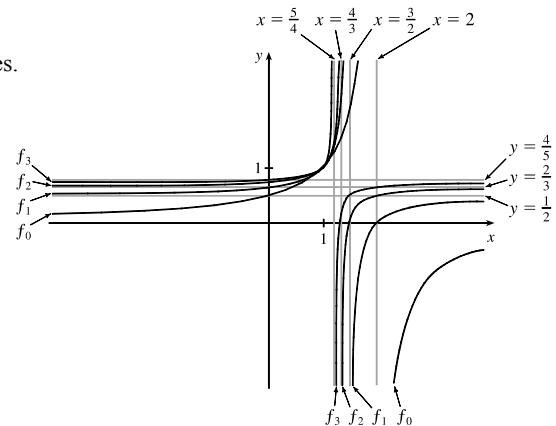
Assume that the formula is true for  $n = k$ ; that is,  $f_k(x) = \frac{k+1-kx}{k+2-(k+1)x}$ . Then

$$\begin{aligned} f_{k+1}(x) &= (f_0 \circ f_k)(x) = f_0(f_k(x)) = f_0\left(\frac{k+1-kx}{k+2-(k+1)x}\right) = \frac{1}{2 - \frac{k+1-kx}{k+2-(k+1)x}} \\ &= \frac{k+2-(k+1)x}{2[k+2-(k+1)x] - (k+1-kx)} = \frac{k+2-(k+1)x}{k+3-(k+2)x} \end{aligned}$$

This shows that the formula for  $f_n$  is true for  $n = k + 1$ . Therefore, by mathematical induction, the formula is true for all positive integers  $n$ .

(b) From the graph, we can make several observations:

- The values at each fixed  $x = a$  keep increasing as  $n$  increases.
- The vertical asymptote gets closer to  $x = 1$  as  $n$  increases.
- The horizontal asymptote gets closer to  $y = 1$  as  $n$  increases.
- The  $x$ -intercept for  $f_{n+1}$  is the value of the vertical asymptote for  $f_n$ .
- The  $y$ -intercept for  $f_n$  is the value of the horizontal asymptote for  $f_{n+1}$ .



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